NATIONAL BUREAU OF STANDARDS REPORT

3481

SOME FRACTIONAL FACTORIAL ARRANGEMENTS
FOR FACTORS AT TWO LEVELS

by

W. H. Clatworthy
W. S. Connor
M. Zelen

<NBS>

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT   **NBS REPORT**

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SOME FRACTIONAL FACTORIAL ARRANGEMENTS
FOR FACTORS AT TWO LEVELS

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W. H. Clatworthy
W. S. Connor
M. Zelen

Statistical Engineering Laboratory

◇NBS◇

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

FOREWORD

This is a technical report on Research in Applications
of Mathematical Statistics to Problems of the Chemical
Corps, for the Biological Laboratories, Chemical Corps,
U. S. Department of the Army carried out in the Statistical
Engineering Laboratory (NBS Project Number 1103-40-5118/52-1).

                                        F.L. Alt
                                        Acting Chief,
                                        National Applied Mathematics
                                                Laboratories

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Director,
National Bureau of Standards

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Some Fractional Factorial Arrangements for
Factors at Two Levels

by

W.H. Clatworthy, W.S. Connor, and M. Zelen

1. Introduction. This report contains 68 experimental
arrangements or plans for fractional replication of factorial
designs of the 2^n series, where n, the number of factors, ranges
from 5 to 12 and each factor is at two levels. The plans include
arrangements which are 1/2, 1/4, 1/8, 1/16, and 1/32 of a full
replication and which involve 8, 16, 32, 64,,128, and 256 measure-
ments. With but two exceptions the experimental units have
been assigned to blocks.

A bibliography which is not exhaustive is given at the
end of this introduction. Several of the designs given in this
report may be found in these references. However this report
represents an independent effort.

A brief description of the plans follows. Each plan has
a designation of the form

Plan n.r.k

which refers to the 1/r replication of the 2^n factorial design
with $2^n/rk$ incomplete blocks of k experimental units each.
For example, Plan 7.2.4 refers to the 1/2 replicate of the 2^7
factorial design with 16 blocks of 4 experimental units each.

In all the designs the n factors, each at two levels, are
designated by capital letters A, B, C, etc. Small
letters refer to assignment of the factors to the experimental
units, the presence of small letter x meaning that the high
level of factor X is applied and the absence of small letter x
meaning that the low level of factor X is applied. For example,
in Plan 5.2.4 the five factors are A, B, C, D, and E. In block
1 of this design one of the experimental units receives the
treatment combination de, i.e., low levels of factors A, B,
and C and high levels of factors D and E are applied to the
experimental unit. The symbol "(1)" is also found in block 1,
indicating that low levels of all factors of the design are
applied to one experimental unit of block 1.

A main effect or interaction is said to be estimable if it
is confounded only with higher order interactions. In any plan
in which a main effect is confounded with two-factor interactions,
it is explicitly stated which two-factor interactions are involved.

For each design there is given a fundamental confounding
relationship composed of groups of capital letters (representing
main effects and interactions) connected with equal signs.
The equal signs are read "is confounded with". The fundamental
confounding relationship is useful in determining how the main
effects and interactions are confounded with each other as a

result of the design being a fractional part of a complete replication. For example, in Plan 6.4.4 the fundamental con-founding relationship is

$$I = ABCE = ABDF = CDEF .$$

To determine how the main effects and interactions are confounded by the 1/4 replication of the full 2^6 factorial, each term of the fundamental confounding relationship is multiplied by the main effect or interaction in question with the understanding that $X^2 = 1$ where $X = A$, B, C, etc. and $I(X...Z) = X ... Z$. For example, the confounding of main effect A is given by

$$A = BCE = BDF = ACDEF .$$

Note that multiplication by BCE, BDF, and ACDEF give exactly the same result, except for order of the groups. The other confounding relationships are:

$$B = ACE = ADF = BCDEF$$
$$C = ABE = ABCDF = DEF$$
$$D = ABCDE = ABF = CEF$$
$$E = ABC = ABDEF = CDF$$
$$F = ABCEF = ABD = CDE$$
$$AB = CE = DF = ABCDEF$$

$$AC = BE = BCDF = ADEF$$

$$AD = BCDE = BF = ACEF$$

$$AE = BC = BDEF = ACDF$$

$$AF = BCEF = BD = ACDE$$

$$CD = ABDE = ABCF = EF$$

$$CF = ABEF = ABCD = DE$$

$$ACD = BDE = BCF = AEF$$

$$ACF = BEF = BCD = ADE$$

The treatment combinations were assigned to the blocks so as to confound interactions AB, BC, and AC with the blocks. If b is the number of blocks, then certain b-1 of the confounding relationships designate which effects cannot be estimated because of block confounding. These are listed for each design as "Block confounding" wherein each group of letters designates a confounding relationship.

Examination of the totality of confounding relationships for Plan 6.4.4 reveals that the main effects A, B, C, D, E, and F are estimable, that the two-factor interactions which are not already confounded with blocks are confounded with other two-factor interactions and hence are not estimable, and that no three-factor interaction is estimable. Thus in this design it is impossible to estimate any interaction unless one is prepared to assume that some other interaction(s) of the same order is(are) negligible.

In some of the plans where the number of treatment combinations is large, the blocks are not written out in full. For such designs only the initial block is recorded in detail and one treatment combination is given for each of the remaining blocks. The other blocks are obtained by successively multiplying the treatment combinations in the initial block by each of the given treatment combinations with the proviso that $x^2 = 1$, (x = a,b,c, etc.). For example, Plan 10.4.8 is such a design. The initial block consists of the following eight treatment combinations

(1), abcd, efhj, abcdefhj, eghk, abcdeghk,

fgjk, abcdfgjk.

To obtain the treatment combinations for block 2, say, one multiplies these eight groups by __ab__ (the treatment combination listed for block 2), obtaining

ab, cd, abefhj, cdefhj, abeghk, cdeghk,

abfgjk, cdfgjk.

Although the plans in this report have been carefully checked, there is always the possibility of errors. The authors would appreciate hearing from anyone discovering errors.

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Plan 5.2.4. 5 factors, 1/2 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E

I = ABCDE

Block confounding: AB, AC, BC.

All main effects and all two-factor interactions except AB, AC, and BC are estimable.

Blocks

1	2	3	4
(1)	ab	ac	ad
abcd	cd	bd	bc
de	abde	acde	ae
abce	ce	be	bcde

Plan 5.2.8. 5 factors, 1/2 replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E

I = ABCDE

Block confounding: AB

All main effects and all two-factor interactions <u>except</u> AB are estimable.

<div align="center">

Blocks

1	2
(1)	ac
abcd	bd
de	acde
abce	be
ab	bc
cd	ad
abde	bcde
ce	ae

</div>

Plan 6.2.4. 6 factors, 1/2 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF, ACF, BC, ABE, EF, BCEF, ACE.

All main effects, all two-factor interactions except AD, BC, and EF are estimable. Twelve of the three-factor interactions are confounded in pairs as follows:

$$ABC = DEF \qquad ADE = BCF$$

$$ABD = CEF \qquad ADF = BCE$$

$$ACD = BEF \qquad AEF = BCD \quad,$$

while the remaining eight three-factor interactions are confounded with blocks.

Blocks

1	2	3	4	5	6	7	8
(1)	ab	ac	bc	ae	be	ce	abce
abcd	cd	bd	ad	bcde	acde	abde	de
bcef	acef	abef	ef	abcf	cf	bf	af
adef	bdef	cdef	abcdef	df	abdf	acdf	bcdf

<u>Plan 6.2.8.</u> 6 factors, 1/2 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF, ACF, BC

All main effects and all two-factor interactions <u>except</u> BC are estimable. Sixteen of the twenty three-factor interactions are confounded in pairs as follows:

ABC = DEF	ACE = BDF
ABD = CEF	ADE = BCF
ABE = CDF	ADF = BCE
ACD = BEF	AEF = BCD ,

while the remaining four three-factor interactions are confounded with blocks.

Blocks

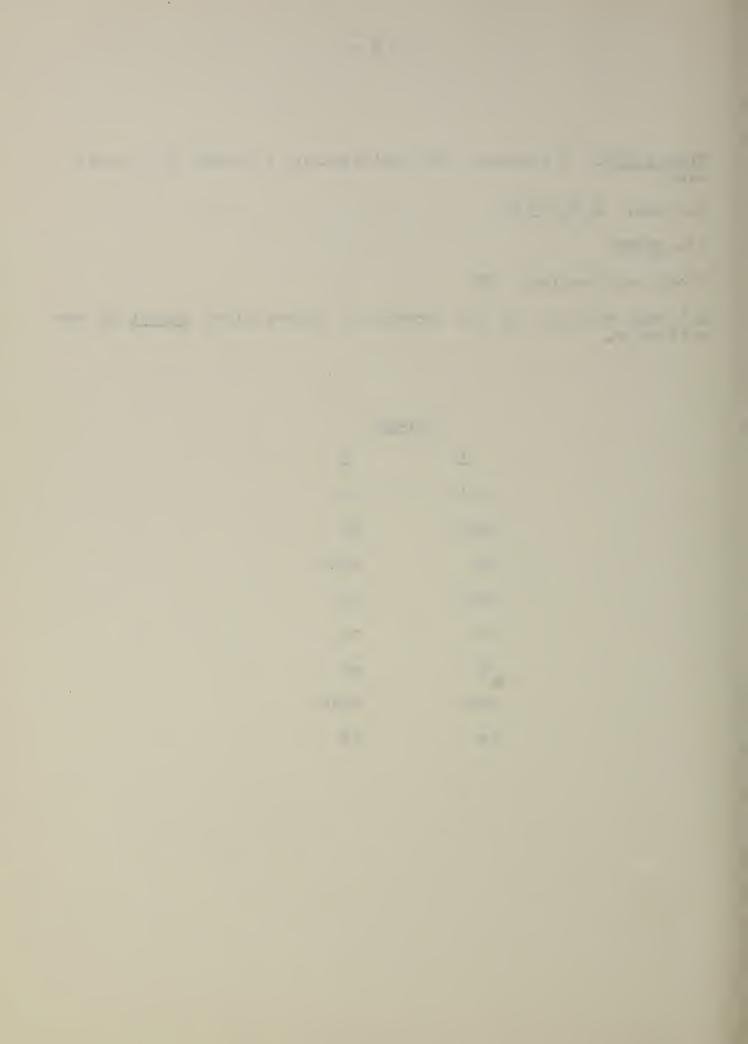| <u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	ab	ac	bc
abcd	cd	bd	ad
bcef	acef	abef	ef
adef	bdef	cdef	abcdef
abce	ce	be	ae
de	abde	acde	bcde
af	bf	cf	abcf
bcdf	acdf	abdf	df

<u>Plan 6.2.16.</u> 6 factors, 1/2 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F

I = ABCDEF

Block confounding: ABF

All main effects and all two-factor interactions are estimable. The three-factor interactions ABF and CDE are confounded with blocks while all others are confounded in pairs as follows:

ABC = DEF	ACD = BEF	ADE = BCF
ABD = CEF	ACE = BDF	ADF = BCE
ABE = CDF	ACF = BDE	AEF = BCD .

Blocks

1		2	
(1)	ab	ac	bc
abcd	cd	bd	ad
bcef	acef	abef	ef
adef	bdef	cdef	abcdef
abce	ce	be	ae
de	abde	acde	bcde
af	bf	cf	abcf
bcdf	acdf	abdf	df

Plan 7.2.4. 7 factors, 1/2 replication, 16 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACFG, BC, ABEG, EF, BCEF, ACEG, ABEF, EG, BCEG, ACEF, FG, AB, AC, BCFG

All main effects, 15 of the 21 two-factor interactions, and 26 of the 35 three-factor interactions are estimable. The two-factor interactions

AB, AC, BC, EF, EG, FG

and the three-factor interactions

ADE, ADF, ADG, BDE, BDF, BDG, CDE, CDF, CDG

are confounded with blocks.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ac	bc	ae	be
abcd	cd	bd	ad	bcde	acde
defg	abdefg	acdefg	bcdefg	adfg	bdfg
abcefg	cefg	befg	aefg	bcfg	acfg

<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	
ce	abce	af	bf	cf	
abde	de	bcdf	acdf	abdf	
cdfg	abcdfg	adeg	bdeg	cdeg	
abfg	fg	bceg	aceg	abeg	

<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>	
abcf	ef	abef	acef	bcef	
df	abcdef	cdef	bdef	adef	
abcdeg	dg	abdg	acdg	bcdg	
eg	abcg	cg	bg	ag	

Plan 7.2.8. 7 factors, 1/2 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACF, BCG, BCEF, ACEG, ABE, EFG

All main effects, all two-factor interactions, and all three-factor interactions except

    ABE, ACF, ADG, BCG, BDF, CDE, and EFG

are estimable.

Blocks

1	2	3	4	5	7	6	8
(1)	ab	ac	bc	ae	ce	be	abce
abcd	cd	bd	ad	bcde	abde	acde	de
bcef	acef	abef	ef	abcf	bf	cf	af
adef	bdef	cdef	abcdef	df	acdf	abdf	bcdf
cdfg	abcdfg	adfg	bdfg	acdefg	defg	bcdefg	abdefg
abfg	fg	bcfg	acfg	befg	abcefg	aefg	cefg
bdeg	adeg	abcdeg	cdeg	abdg	bcdg	dg	acdg
aceg	bceg	eg	abeg	cg	ag	abcg	bg

Plan 7.2.16. 7 factors, 1/2 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG, ACF, BCG

All main effects, all two-factor interactions, and all three-factor interactions except ACF, BCG, and CDE are estimable.

Blocks

1			2	
(1)	abce		ab	ce
abcd	de		cd	abde
bcef	af		acef	bf
adef	bcdf		bdef	acdf
cdfg	abdefg		abcdfg	defg
abfg	cefg		fg	abcefg
bdeg	acdg		adeg	bcdg
aceg	bg		bceg	ag

3			4	
ac	be		bc	ae
bd	acde		ad	bcde
abef	cf		ef	abcf
cdef	abdf		abcdef	df
adfg	bcdefg		bdfg	acdefg
bcfg	aefg		acfg	befg
abcdeg	dg		cdeg	abdg
eg	abcg		abeg	cg

Plan 7.2.32. 7 factors, 1/2 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G

I = ABCDEFG

Block confounding: ABFG

All main effects, all two-factor interactions and all three-factor interactions except CDE are estimable.

Blocks

1

(1)	abce	ab	ce
abcd	de	cd	abde
bcef	af	acef	bf
adef	bcdf	bdef	acdf
cdfg	abdefg	abcdfg	defg
abfg	cefg	fg	abcefg
bdeg	acdg	adeg	bcdg
aceg	bg	bceg	ag

2

ac	be	bc	ae
bd	acde	ad	bcde
abef	cf	ef	abcf
cdef	abdf	abcdef	df
adfg	bcdefg	bdfg	acdefg
bcfg	aefg	acfg	befg
abcdeg	dg	cdeg	abdg
eg	abcg	abeg	cg

<u>Plan 8.2.8.</u> 8 factors, 1/2 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABCE, DE, ABDEG, CEG, CDG, ABG, BCH, ADH, AEH, BCDEH, ACDEGH, BEGH, BDGH, ACGH.

All main effects, all two-factor interactions <u>except</u> BF and DE, and all three-factor interactions <u>except</u> ABG, <u>ADH</u>, AEH, BCH, CDG, and CEG are estimable. None of the four-factor interactions is estimable.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ac	bc	ad	bd
abfh	fh	bcfh	acfh	bdfh	adfh
bcfg	acfg	abfg	fg	abcdfg	cdfg
acgh	bcgh	gh	abgh	cdgh	abcdgh
cdeh	abcdeh	adeh	bdeh	aceh	bceh
abcdef	cdef	bdef	adef	bcef	acef
bdefgh	adefgh	abcdefgh	cdefgh	abefgh	efgh
adeg	bdeg	cdeg	abcdeg	eg	abeg

<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>
cd	abcd	ae	be	ce
abcdfh	cdfh	befh	aefh	abcefh
bdfg	adfg	abcefg	cefg	befg
adgh	bdgh	cegh	abcegh	aegh
eh	abeh	acdh	bcdh	dh
abef	ef	bcdf	acdf	abdf
bcefgh	acefgh	abdfgh	dfgh	bcdfgh
aceg	bceg	dg	abdg	acdg

Plan 8.2.8. (Continued).

Blocks

12	13	14	15	16
abce	de	abde	acde	bcde
cefh	abdefh	defh	bcdefh	acdefh
aefg	bcdefg	acdefg	abdefg	defg
begh	acdegh	bcdegh	degh	abdegh
abdh	ch	abch	ah	bh
df	abcf	cf	bf	af
acdfgh	bfgh	afgh	abcfgh	cfgh
bcdg	ag	bg	cg	abcg

Plan 8.2.16. 8 factors, 1/2 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABEF, CDEF, BCEG, ADEG, ACFG, BDFG.

All main effects, all two-factor interactions, and all three-factor interactions are estimable. Fourteen of the 70 four-factor interactions are confounded with blocks while the remaining 56 are confounded in pairs.

Blocks

1	2	3	4
(1)	ab	ac	bc
abcd	cd	bd	ad
abef	ef	bcef	acef
cdef	abcdef	adef	bdef
bceg	aceg	abeg	eg
adeg	bdeg	cdeg	abcdeg
acfg	bcfg	fg	abfg
bdfg	adfg	abcdfg	cdfg
efgh	abefgh	acefgh	bcefgh
abcdefgh	cdefgh	bdefgh	adefgh
abgh	gh	bcgh	acgh
cdgh	abcdgh	adgh	bdgh
bcfh	acfh	abfh	fh
adfh	bdfh	cdfh	abcdfh
aceh	bceh	eh	abeh
bdeh	adeh	abcdeh	cdeh

Plan 8.2.16.   (Continued)

Blocks

5	6	7	8
ae	be	ce	abce
bcde	acde	abde	de
bf	af	abcf	cf
acdf	bcdf	df	abdf
abcg	cg	bg	ag
dg	abdg	acdg	bcdg
cefg	abcefg	aefg	befg
abdefg	defg	bcdefg	acdefg
afgh	bfgh	cfgh	abcfgh
bcdfgh	acdfgh	abdfgh	dfgh
begh	aegh	abcegh	cegh
acdegh	bcdegh	degh	abdegh
abcefh	cefh	befh	aefh
defh	abdefh	acdefh	bcdefh
ch	abch	ah	bh
abdh	dh	bcdh	acdh

Plan 8.2.32. 8 factors, 1/2 replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD, ABEF, CDEF

All main effects, all two-factor interactions, and all three-factor interactions are estimable. Six of the 70 four-factor interactions are confounded with blocks and the other 64 are confounded in pairs.

Blocks

Combine blocks in Plan 8.2.16 as follows:

$$1 \text{ and } 2$$

$$3 \text{ and } 4$$

$$5 \text{ and } 6$$

$$7 \text{ and } 8 \ .$$

Plan 8.2.64. 8 factors, 1/2 replication, 2 blocks of 64 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCDEFGH

Block confounding: ABCD

All main effects, all two-factor and all three-factor inter-actions are estimable. Two of the 70 four-factor interactions are confounded with blocks while the remaining 68 are confounded in pairs.
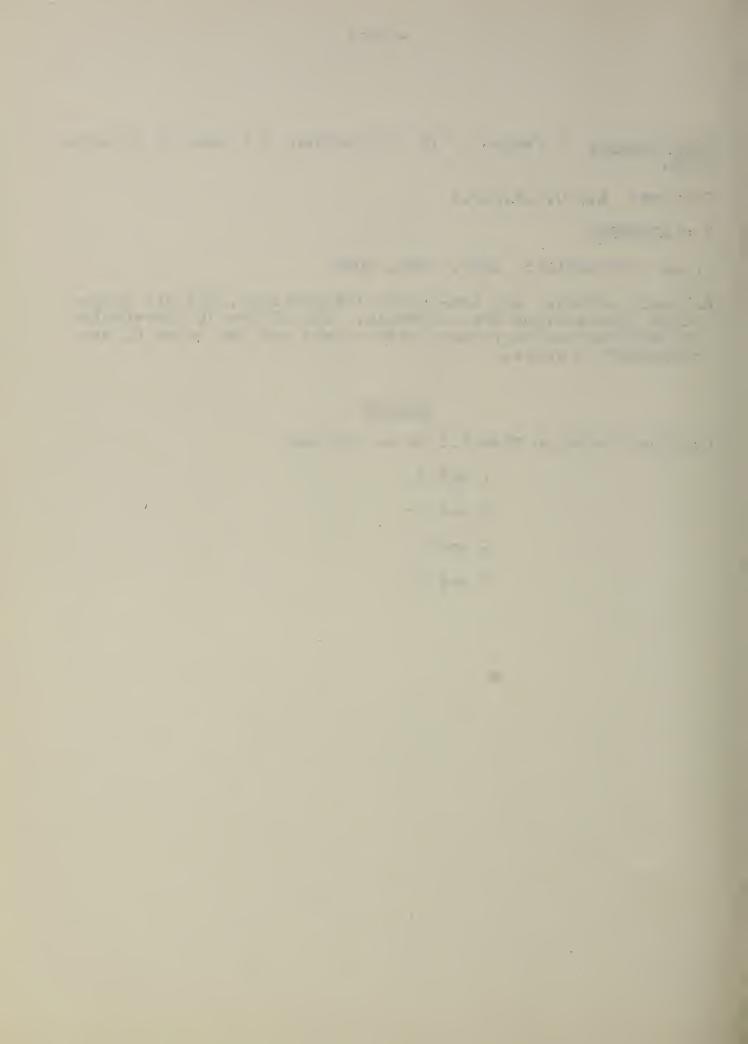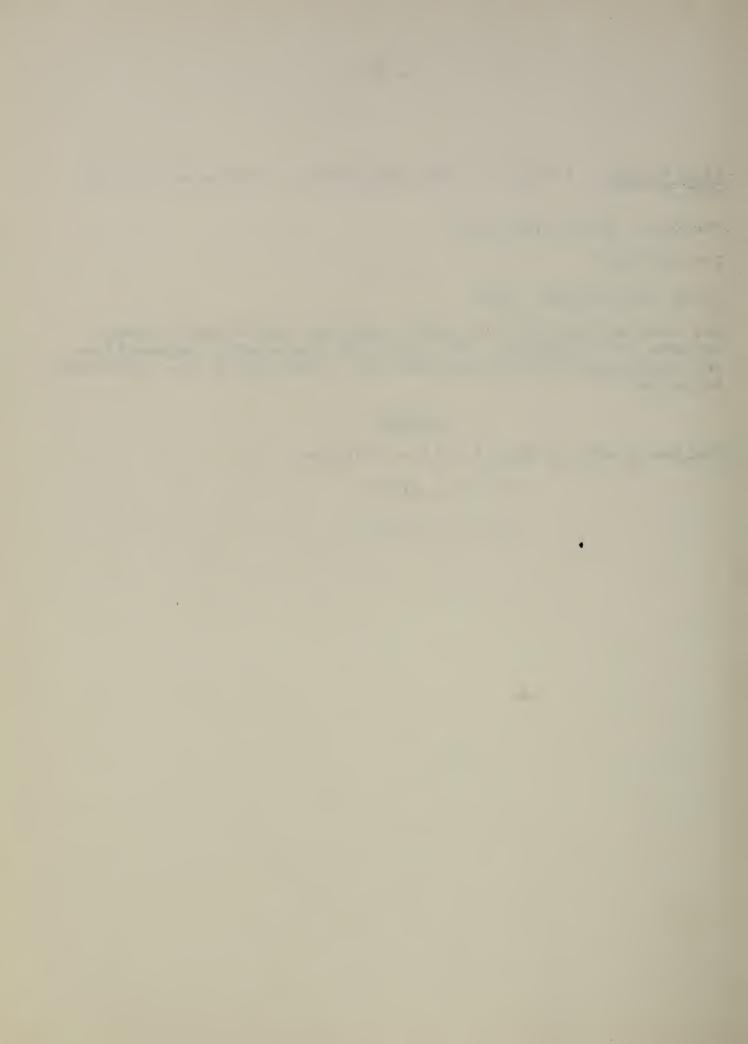
Blocks

Combine blocks in Plan 8.2.16 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

- 22 -

Plan 9.2.8. 9 factors, 1/2 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABFGJ, ACFG, BCJ, ADG, BDFJ, CDF, ABCDGJ,

FGHJ, ABH, ACHJ, BCFGH, ADFHJ, BDGH, CDGHJ,

ABCDFH, ACE, BCEFGJ, EFG, ABEJ, CDEG, ABCDEFJ,

ADEF, BDEGJ, ACEFGHJ, BCEH, EHJ, ABEFGH,

CDEFHJ, ABCDEGH, ADEGHJ, BDEFH.

The main effects, 33 of the 36 two-factor interactions, 71 of the 84 three-factor interactions and 111 of the 126 four-factor interactions are estimable. The following interactions are confounded with blocks:

BD, FJ, GH, ABG, ABH, ACE, ADG, ADH, BCF, BCJ,

CDF, CDJ, EFG, EFH, EGJ, EHJ, ABEF, ABEJ, ACFG,

ACFH,,ACGJ, ACHJ, ADEF, ADEJ, BCEG, BCEH, BDFJ,

BDGH, CDEG, CDEH, FGHJ.

Plan 9.2.8. (Continued).

Blocks

1.(Initial Block)	Block Multipliers						
(1)	2	3	4	5	6	7	
abcd	ab	ac	bc	ae	be	ce	
aegh							
bcdegh	8	9	10	11	12	13	
cefj	abce	af	bf	cf	abcf	ef	
abdefj							
acfghj	14	15	16	17	18	19	
bdfghj	abef	acef	bcef	ag	bg	cg	
	20	21	22	23	24	25	
	abcg	eg	abeg	aceg	bceg	fg	
	26	27	28	29	30	31	32
	abfg	acfg	bcfg	aefg	befg	cefg	abcefg

Plan 9.2.16. 9 factors, 1/2 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABCDE, ABCF, DEF, BDEFGH, ACFGH, ACDEGH, BGH, BCEH, ADH, AEFH, BCDFH, CDFG, ABEFG, ABDG, CEG.

All main effects, all two-factor interactions, all three-factor interactions <u>except</u>

$$\text{ACJ, ADH, BFJ, BGH, CEG, and DEF}$$

and all four-factor interactions <u>except</u>

$$\text{ABCF, ABDG, AEFH, AEGJ, BCEH, BDEJ, CDFG, CDHJ, FGHJ}$$

are estimable.

Blocks

1		2	3	4	5	6
(1)	befg	ab	ac	bc	ad	bd
acde	abcdfg					
adfj	abdegj					
cefj	bcgj	7	8	9	10	11
abhj	aefghj	cd	abcd	af	bf	cf
bcdehj	cdfghj					
bdfh	degh					
abcefh	acgh	12	13	14	15	16
		abcf	df	abdf	acdf	bcdf

Plan 9.2.32. 9 factors, 1/2 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCDEFGHJ

Block confounding: ABCD, CDEF, ABEF, ACEG, BDEG, ADFG, BCFG

The main effects, all two-factor interactions, all three-factor interactions, and all four-factor interactions <u>except</u> those mentioned above are estimable.

Blocks

1. (Initial Block)

(1)	hj
abcd	abcdhj
abef	abefhj
cdef	cdefhj
bdeg	bdeghj
aceg	aceghj
adfg	adfghj
bcfg	bcfghj
bceh	bcej
adeh	adej
acfh	acfj
bdfh	bdfj
cdgh	cdgj
abgh	abgj
abcdefgh	abcdefgj
efgh	efgj

Block Multipliers

2. ab
3. ac
4. bc
5. ae
6. be
7. ce
8. abce

Plan 6.4.2. 6 factors, 1/4 replication, 8 blocks of 2 units each.

Factors: A,B,C,D,E,F

I = ABCE = ABDF = CDEF

Block confounding: AB, BC, AC, EF, ABEF, BCEF, ACEF

All main effects are estimable. All two-factor interactions have two-and four-factor interactions as aliases. Three-factor interactions are aliases of main effects or of other three-factor interactions:

$$ACD = BDE = BCF = AEF$$

$$ACF = BEF = BCD = ADE \, .$$

Blocks

1	2	3	4
(1)	ab	ce	abce
abcdef	cdef	abdf	df

5	6	7	8
acd	bcd	ade	bde
bef	aef	bcf	acf

Plan 6.4.4. 6 factors, 1/4 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E,F

I = ABCE = ABDF = CDEF

Block confounding: AB, BC, AC

The main effects are estimable, but all two factor-interactions have two- and four-factor aliases while all three-factor interactions are aliases of either main effects or three-factor interactions.

Blocks

1	2	3	4
(1)	ab	acd	bcd
abcdef	cdef	bef	aef
abce	ce	bde	ade
df	abdf	acf	bcf

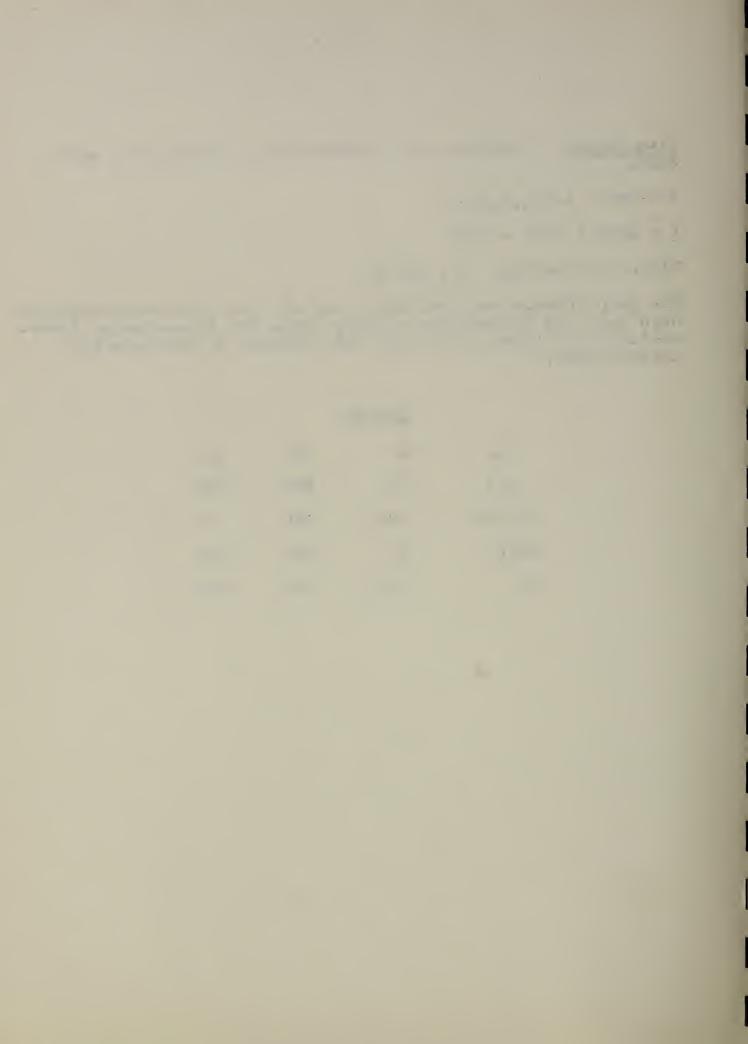Plan 6.4.8. 6 factors, 1/4 replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E,F

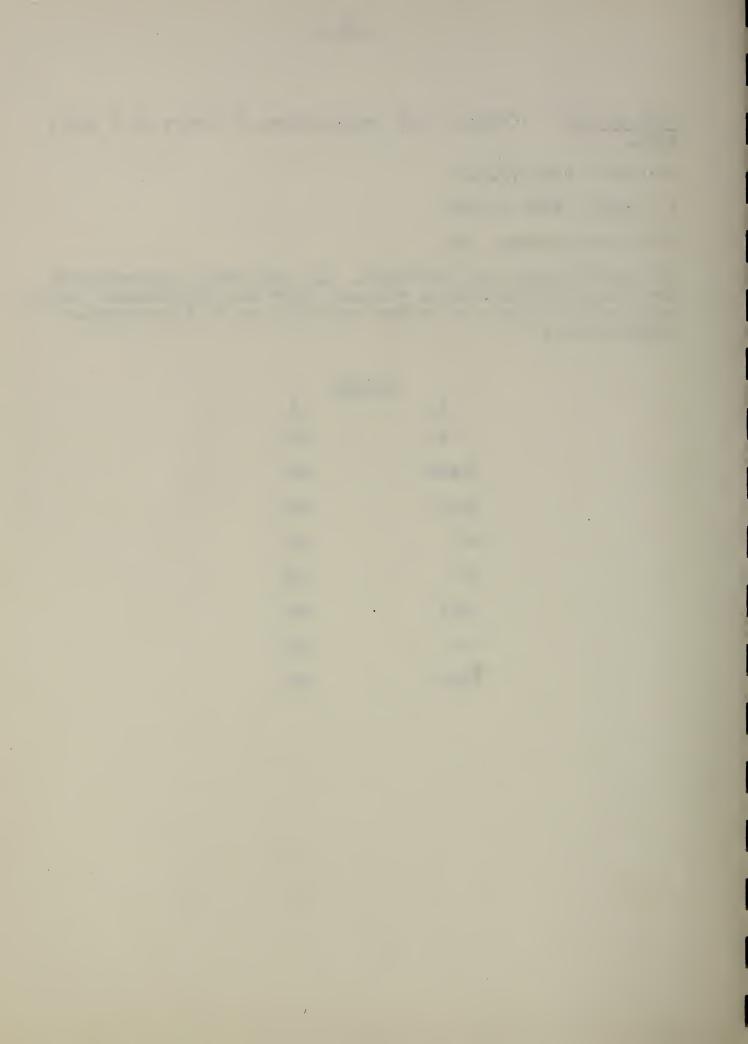I = ABCE = ABDF = CDEF

Block confounding: AB

The main effects are estimable. All two-factor interactions have two- and four-factor aliases while all three-factor inter-actions are aliases of either main effects or three-factor interactions.

Blocks	
1	2
(1)	acd
abcdef	bef
abce	bde
df	acf
ab	bcd
cdef	aef
ce	ade
abdf	bcf

<u>Plan 7.4.4.</u> 7 factors, 1/4 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCE = ABDFG = CDEFG

Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF

The main effects, 12 of the 21 two-factor interactions, and 5 of the higher order interactions are estimable. The following two-factor interactions are estimable:

AD, AF, AG, BD, BF, BG, CD, CF, CG, DE, EF, EG.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
(1)	ab	df	abdf	dg	abdg	fg	abfg
abce	ce	abcdef	cdef	abcdeg	cdeg	abcefg	cefg
adefg	bdefg	aeg	beg	aef	bef	ade	bde
bcdfg	acdfg	bcg	acg	bcf	acf	bcd	acd

Plan 7.4.8. 7 factors, 1/4 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCEG = ABDF = CDEFG

Block confounding: AB, AD, BD

The main effects and all of the two-factor interactions except (for those already confounded in pairs by the fundamental identity)

            AB, AD, AF, BD, BF, DF

are estimable. Six of the three-factor interactions have as aliases six other three-factor interactions.

Blocks

1	2	3	4
(1)	ab	bcd	acd
ce	abce	bde	ade
cg	abcg	bdg	adg
eg	abeg	bcdeg	acdeg
abdf	df	acf	bcf
abcdef	cdef	aef	bef
abcdfg	cdfg	afg	bfg
abdefg	defg	acefg	bcefg

Plan 7.4.16.   7 factors, 1/4 replication, 2 blocks of 16 units each.
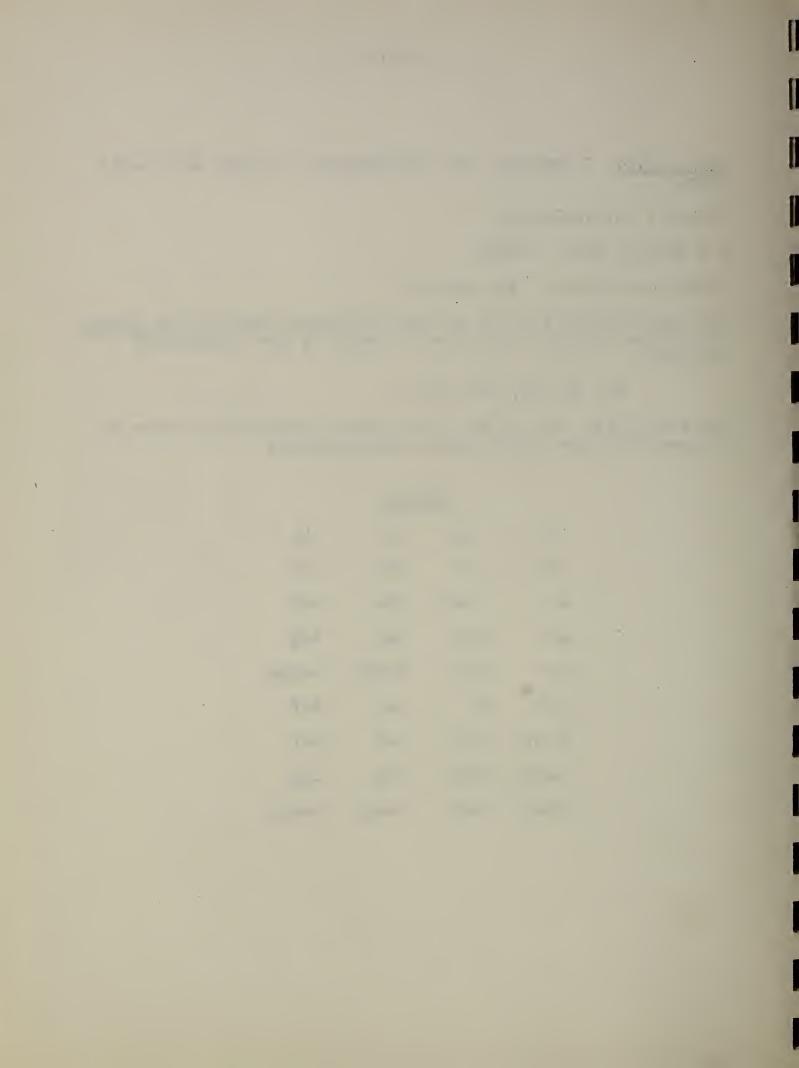
Factors: A,B,C,D,E,F,G

I = ABCEG = ABDF = CDEFG

Block confounding: AB

All main effects and all two-factor interactions <u>except</u>
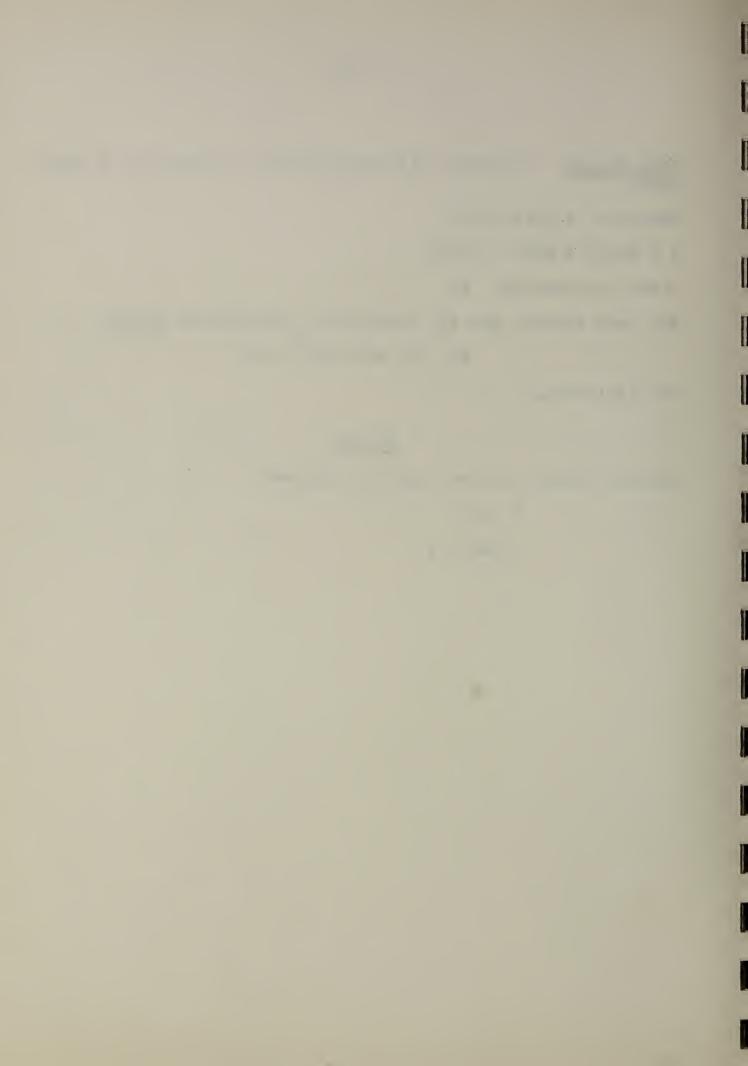
AB, AD, AF, BD, BF, DF

are estimable.

<u>Blocks</u>

Combine blocks in Plan 7.4.8 as follows:

1 and 2

3 and 4 .

Plan 8.4.4. 8 factors, 1/4 replication, 16 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF, DE, ACE, BDF, ABCF, BCDE, ABE, CDF, AF

The main effects, all two-factor interactions <u>except</u>

AF, AH, BC, BG, CG, DE, FH ,

and 19 of the higher order interactions are estimable.

<u>Blocks</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	acd	bcd	aef	bef
adefh	bdefh	cefh	abcefh	dh	abdh
bcdeg	acdeg	abeg	eg	abcdfg	cdfg
abcfgh	cfgh	bdfgh	adfgh	bcegh	acegh

<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>
cdef	abcdef	agh	bgh	cdgh
ach	bch	defg	abdefg	acefg
bfg	afg	abcdeh	cdeh	beh
abdegh	degh	bcf	acf	abdf

<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
abcdgh	efgh	abefgh	acdefgh	bcdefgh
bcefg	adg	bdg	cg	abcg
aeh	bcdfh	acdfh	abfh	fh
df	abce	ce	bde	ade

<u>Plan 8.4.8.</u>   8 factors, 1/4 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

Block confounding: ACD, BEF, ABCDEF, BC, ABD, CEF, ADEF

The main effects, all two-factor interactions <u>except</u> BC and FH, and 22 higher order interactions are estimable.

<u>Blocks</u>

Combine blocks in Plan 8.4.4 as follows:

1 and 13	5 and 9
2 and 14	6 and 10
3 and 15	7 and 11
4 and 16	8 and 12 .

Plan 8.4.16. 8 factors, 1/4 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCEG = ABDFH = CDEFGH

Block confounding: ACD, BEF, ABCDEF

The main effects, all two-factor interactions, and 24 higher order interactions are estimable.

Blocks

Combine blocks in Plan 8.4.4 as follows:
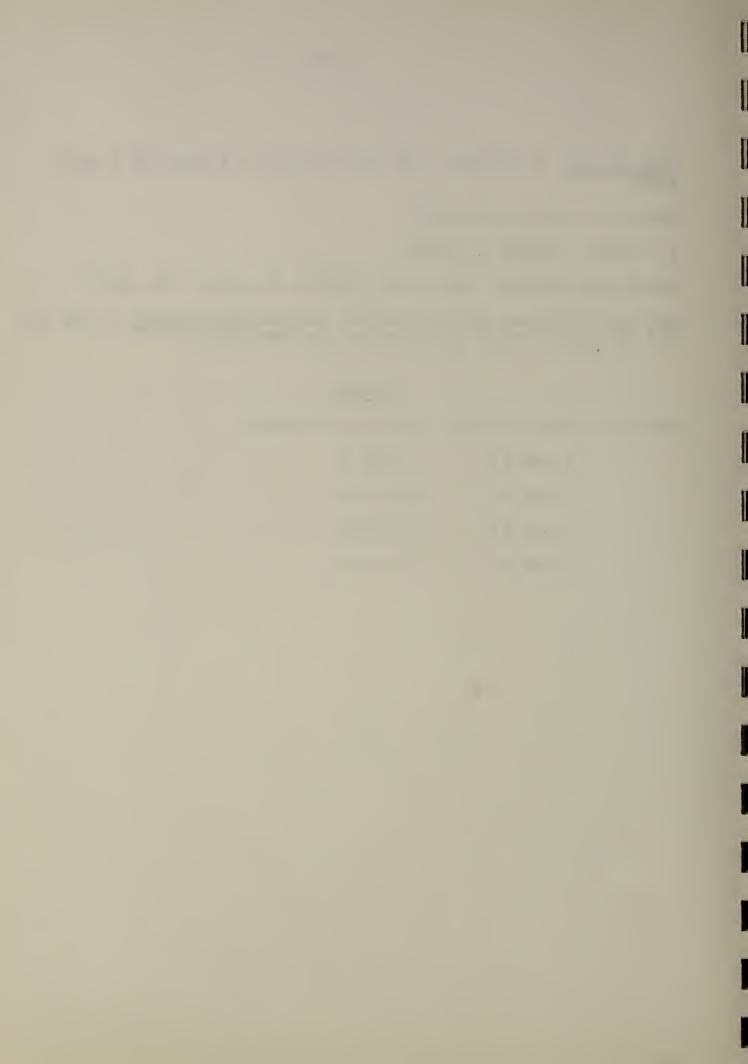
1, 7, 11, and 13   3, 5, 9, and 15

2, 8, 12, and 14   4, 6, 10, and 16.

Plan 8.4.32. 8 factors, 1/4 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H
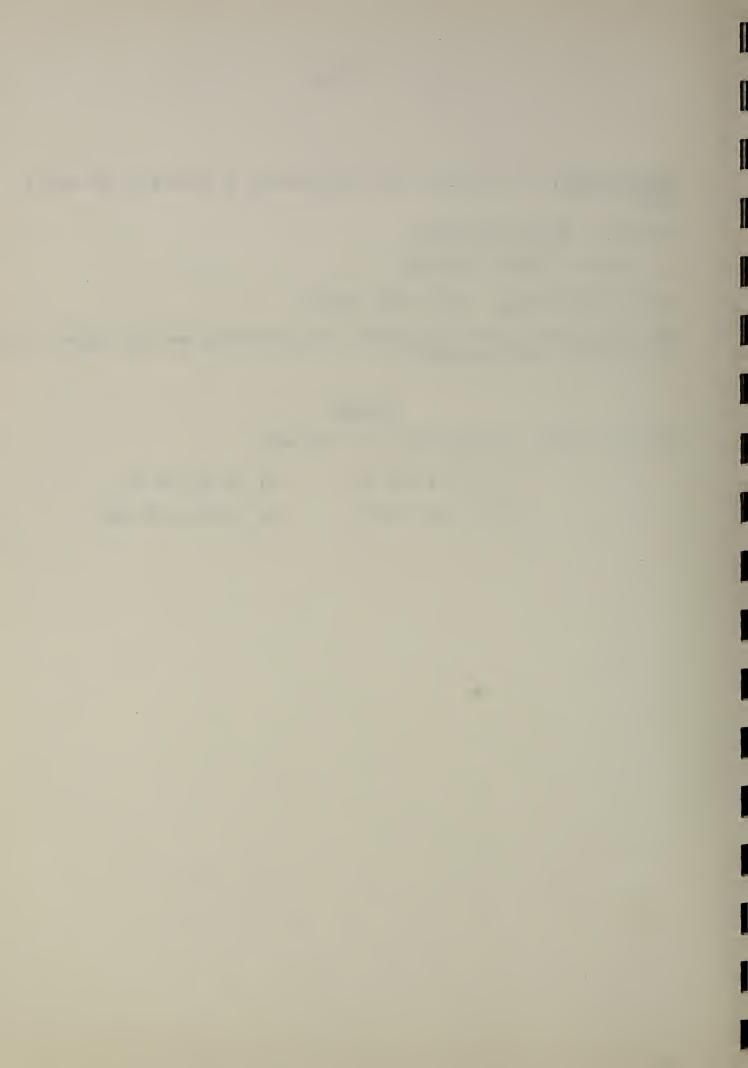
I = ABCEG = ABDFH = CDEFGH
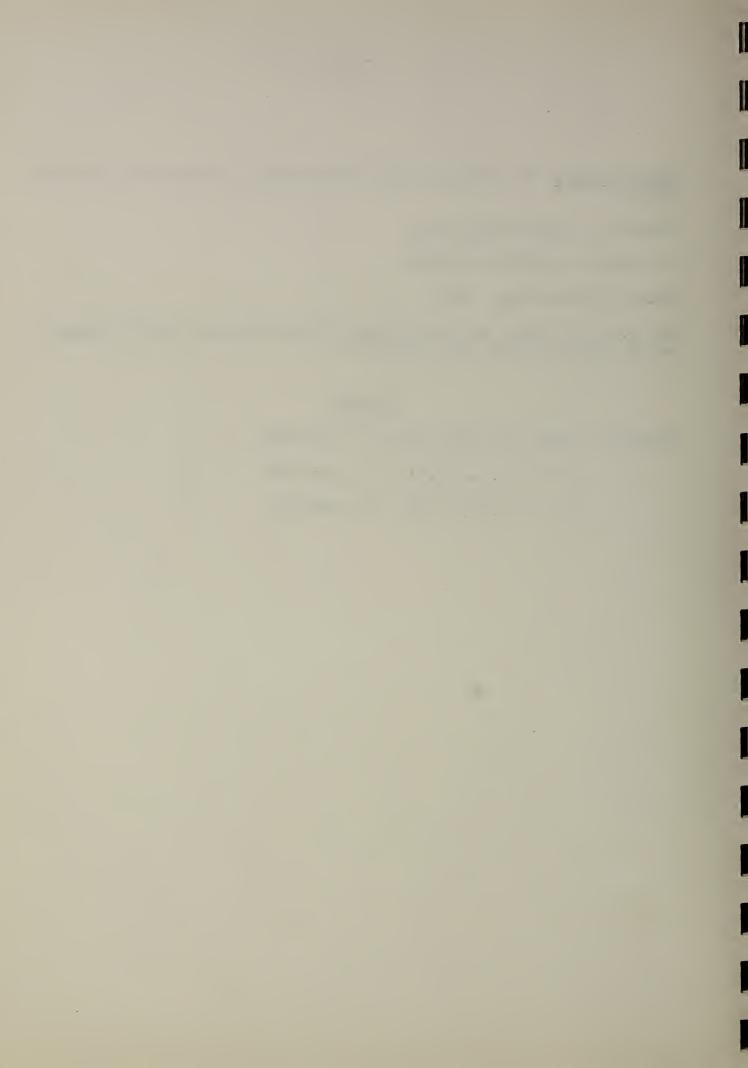
Block confounding: ACD

The main effects, all two-factor interactions, and 26 higher order interactions are estimable.

Blocks

Combine blocks in Plan 8.4.4 as follows:

1, 4, 6, 7, 10, 11, 13, and 16

2, 3, 5, 8, 9, 12, 14, and 15.

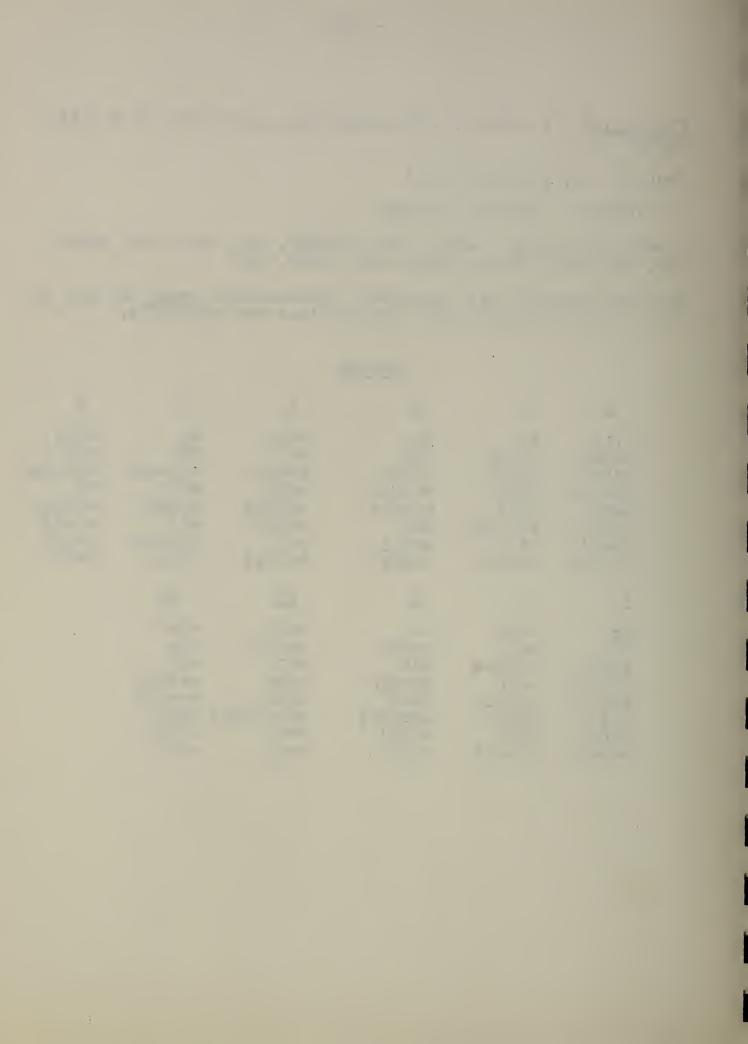Plan 9.4.8. 9 factors, 1/4 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF, BCJ, ABD, CEF, ADEFJ, DEJ, ACE, BDF, ABCFJ, BCDE, ABEJ, CDFJ, AF.

The main effects, all two-factor interactions except AF and CG, and 69 of the higher order interactions are estimable.

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
(1)	ab	ce	abce	cg	abcg
bcdeg	acdeg	bdg	adg	bde	ade
adefh	bdefh	acdfh	bcdfh	acdefgh	bcdefgh
abcfgh	cfgh	abefgh	efgh	abfh	fh
bdhj	adhj	bcdehj	acdehj	bcdghj	acdghj
ceghj	abceghj	ghj	abghj	ehj	abehj
abefj	efj	abcfj	cfj	abcefgj	cefgj
acdfgj	bcdfgj	adefgj	bdefgj	adfj	bdfj

<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>
eg	abeg	df	abdf	cdef
bcd	acd	bcefg	acefg	bfg
adfgh	bdfgh	aeh	beh	ach
abcefh	cefh	abcdgh	cdgh	abdegh
bdeghj	adeghj	bfhj	afhj	bcefhj
chj	abchj	cdefghj	abcdefghj	dfghj
abfgj	fgj	abdej	dej	abcdj
acdefj	bcdefj	acgj	bcgj	aegj

Plan 9.4.8. (Continued)

Blocks

12	13	14	15	16
abcdef	cdfg	abcdfg	defg	abdefg
afg	bef	aef	bcf	acf
bch	acegh	bcegh	agh	bgh
degh	abdh	dh	abcdeh	cdeh
acefhj	bcfghj	acfghj	befghj	aefghj
abdfghj	defhj	abdefhj	cdfhj	abcdfhj
cdj	abcdegj	cdegj	abdgj	dgj
begj	aj	bj	acej	bcej

Plan 9.4.16. 9 factors, 1/4 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF, BCJ, ABD, CEF, ADEFJ.

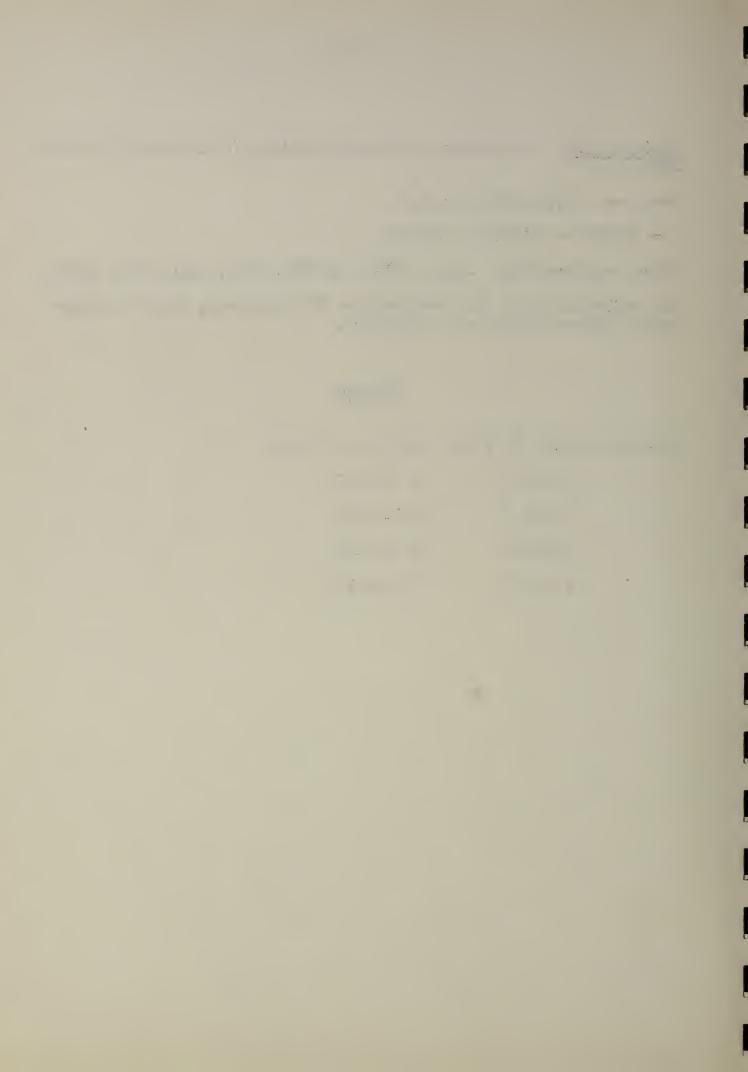The main effects, all two-factor interactions, and 75 higher order interactions are estimable.

Blocks

Combine blocks in Plan 9.4.8 as follows:

1 and 4	9 and 12
2 and 3	10 and 11
5 and 8	13 and 16
6 and 7	14 and 15 .

Plan 9.4.32.   9 factors, 1/4 replication, 4 blocks of 32 units each.

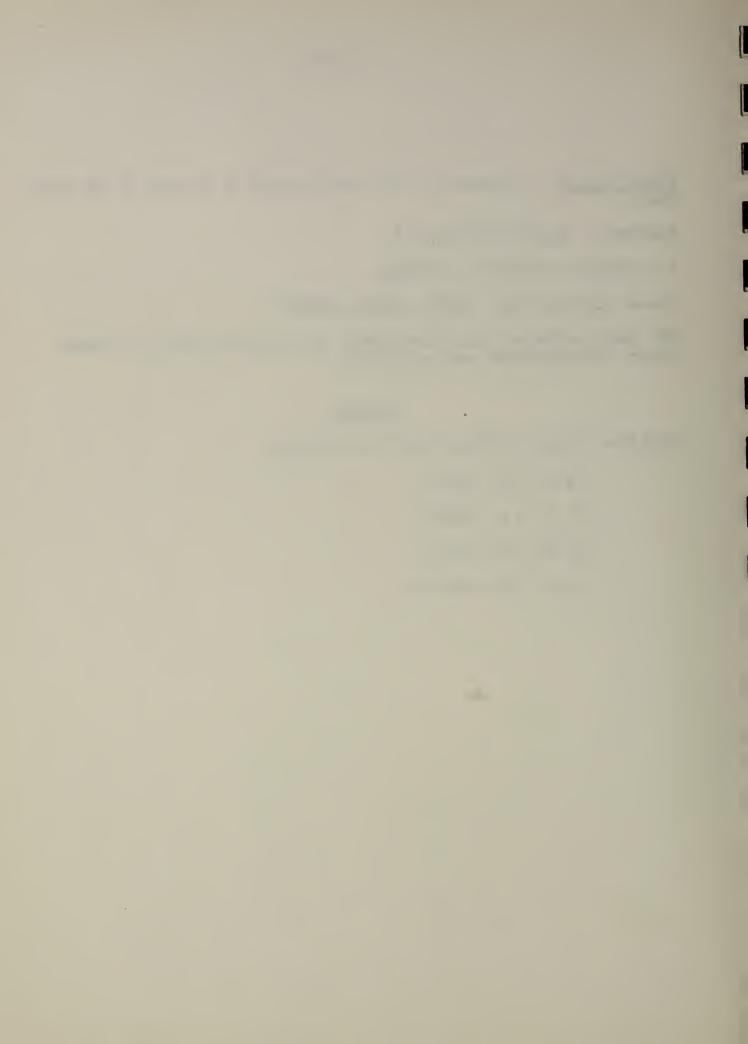Factors: A,B,C,D,E,F,G,H,J

I = ABCEGJ = ABDFHJ = CDEFGH

Block confounding: ACDJ, BEFJ, ABCDEF

All main effects, all two-factor interactions, and 79 higher order interactions are estimable.

Blocks

Combine blocks in Plan 9.4.8 as follows:

       1, 4, 10, and 11

       2, 3, 9, and 12

       5, 8, 14, and 15

       6, 7, 13, and 16.

Plan 10.4.8. 10 factors, 1/4 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDEFG = ABCDHJK = EFGHJK

Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH, ACGK, BCEFGHJK, ADEFGHJK, BDGK, EGHK, ABFGJK, CDFGJK, ABCDEGHK, ACFJ, BCEH, ADEH, BDFJ, EFHJ, AB, CD, ABCDEFHJ, FGJK, ABEGHK, CDEGHK, ABCDFGJK, ACEFGHJK, BCGK, ADGK, BDEFGHJK

All main effects, 36 of the 45 two-factor interactions, and 178 higher order interactions are estimable. The following two-factor interactions are confounded with blocks:

AB, AC, AD, BC, BD, CD, EH, FJ, and GK.

Blocks

1	2	3	4	5	6	7	8
(1)	ab	ac	bc	ef	abef	acef	bcef
abcd							
efhj							
abcdefhj	9	10	11	12	13	14	
eghk							
abcdeghk	eg	abeg	aceg	bceg	fg	abfg	
fgjk							
abcdfgjk						20	
	15	16	17	18	19		
	acfg	bcfg	aeh	beh	ceh	abceh	
	21	22	23	24	25	26	
	afh	bfh	cfh	abcfh	agh	bgh	
	27	28	29	30	31	32	
	cgh	abcgh	aefgh	befgh	cefgh	abcefgh	

Plan 10.4.16. 10 factors, 1/4 replication, 16 blocks of 16 units each.
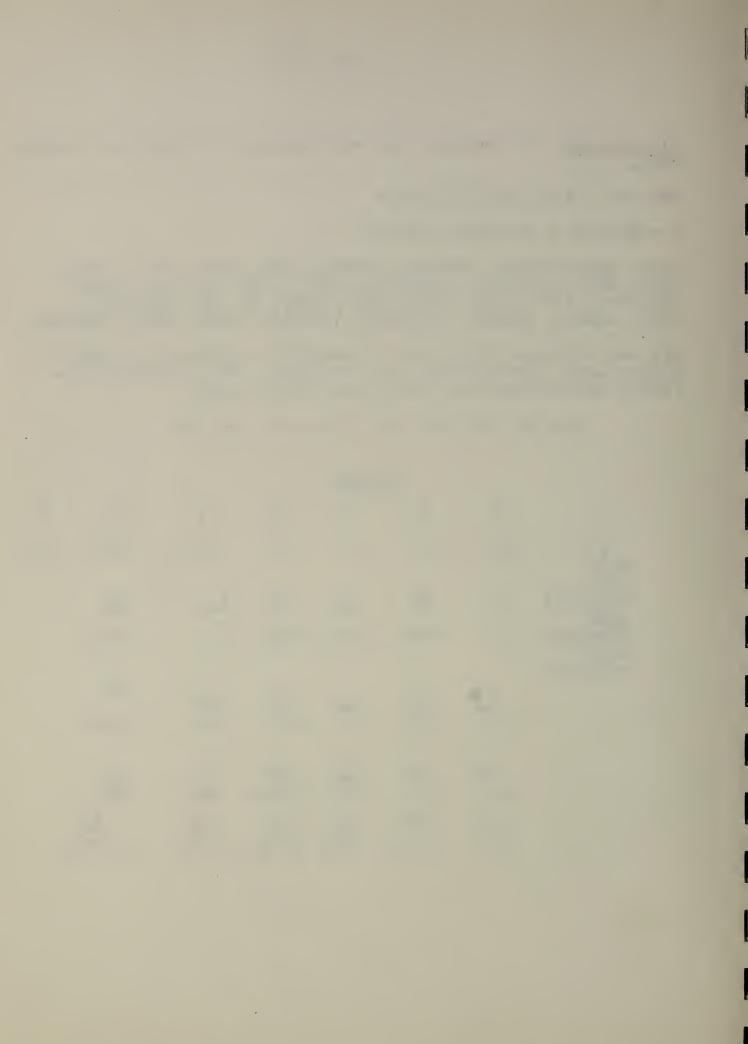
Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDEFG = ABCDHJK = EFGHJK

Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH, ACGK, BCEFGHJK, ADEFGHJK, BDGK, EGHK, ABFGJK, CDFGJK, ABCDEGHK.

All main effects, 42 of the 45 two-factor interactions, and 188 higher order interactions are estimable. Interactions AD, BC, and FJ are confounded with blocks.

Blocks

Combine blocks in Plan 10.4.8 as follows:

1 and 12	17 and 28
2 and 11	18 and 27
3 and 10	19 and 26
4 and 9	20 and 25
5 and 16	21 and 32
6 and 15	22 and 31
7 and 14	23 and 30
8 and 13	24 and 29 .

Plan 10.4.32.   10 factors, 1/4 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDEFG = ABCDHJK = EFGHJK

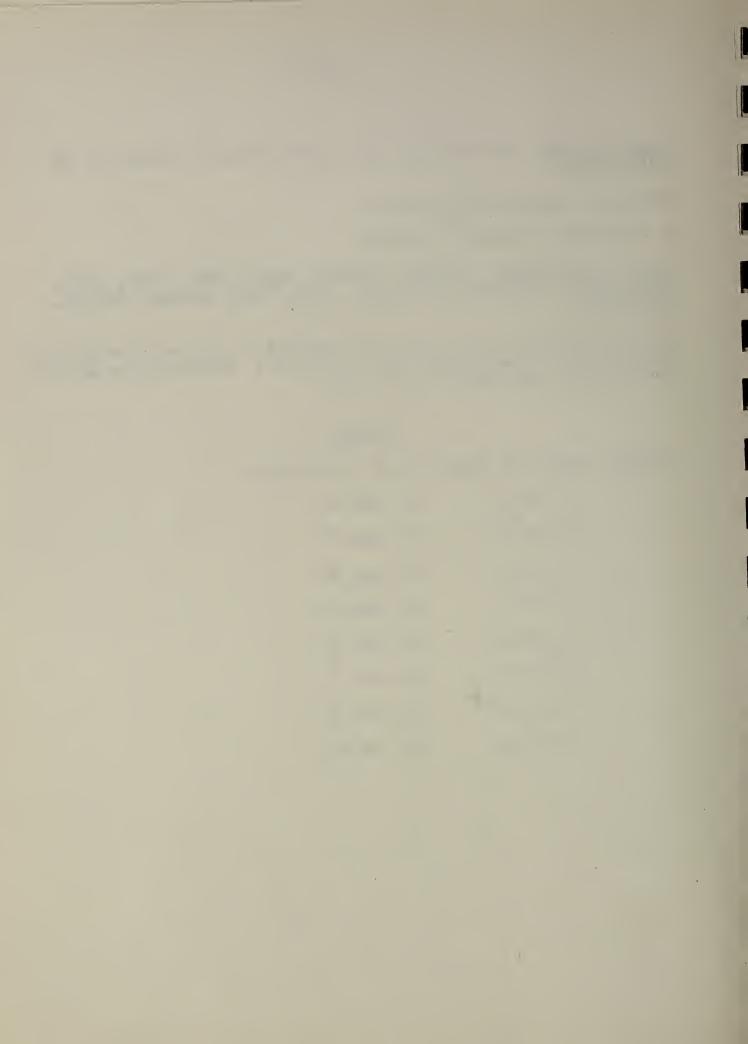Block confounding: ABEFHJ, CDEFHJ, ABCD, ACEH, BCFJ, ADFJ, BDEH.

All main effects, all two-factor interactions, and 193 higher order interactions are estimable.

Blocks

Combine blocks in Plan 10.4.8 as follows:

<div align="center">

1, 6, 12, and 15

2, 5, 11, and 16

3, 8, 10, and 13

4, 7, 9, and 14

17, 22, 28, and 31

18, 21, 27, and 32

19, 24, 26, and 29

20, 23, 25, and 30.

</div>

Plan 8.8.4. 8 factors, 1/8 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH

Block confounding: EGH, FG, EFH, BEH, BG, BEFGH, BF

All main effects and the following two-factor interactions are estimable:

AE, AH, BE, BH, CE, CH, DE, DH, EF,

EG, EH, FH, GH

Blocks

1	2	3	4	5	6	7	8
(1)	abcd	acefgh	bdefgh	cdf	abf	adegh	bcegh
abcdefg	efg	bdh	ach	abeg	cdeg	bcfh	adfh
eh	abcdeh	acfg	ace	cdefh	abefh	adg	bcg
abcdfgh	fgh	bde	bdfg	abgh	cdgh	bcef	adef

<u>Plan 8.8.8.</u> 8 factors, 1/8 replication, 4 blocks of 8 units each.

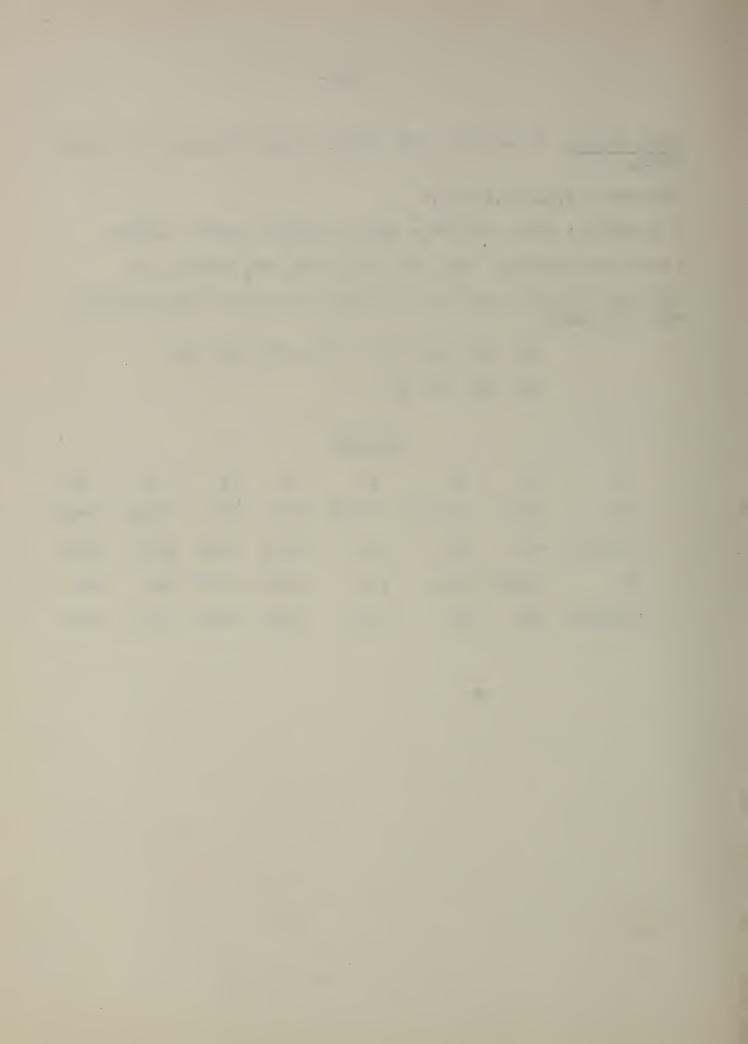Factors: A,B,C,D,E,F,G,H

I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH

Block confounding: EGH, FG, EFH

All main effects and the following two-factor interactions are estimable:

AE, AH, BE, BH, CE, CH, DE, DH, EF, EG, EH, FH, GH .

The following two-factor interactions are confounded with one another:

AF = CG, AG = CF, BF = DG, BG = DF .

<div align="center">

Blocks

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	acefgh	cdf	adegh
abcdefg	bdh	abeg	bcfh
eh	acfg	cdefh	adg
abcdfgh	bde	abgh	bcef
abcd	bdefgh	abf	bcegh
efg	ach	cdeg	adfh
abcdeh	ace	abefh	bcg
fgh	bdfg	cdgh	adef

</div>

Plan 8.8.16. 8 factors, 1/8 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H

I = ABEGH = ACFG = BCEFH = ABCD = CDEGH = BDFG = ADEFH

Block confounding: EGH

All main effects are estimable and the following two-factor interactions are estimable:

AE, AH , BE, BH, CE, CH, DE, DH, EF, EG, EH, FH, GH .

The following two-factor interactions are confounded with one another:

AF=CG, AG=CF, BF=DG, BG=DF, AD=BC .

Blocks

1	2
(1)	cdf
abcdefg	abeg
eh	cdefh
abcdfgh	abgh
abcd	abf
efg	cdeg
abcdeh	abefh
fgh	cdgh
acefgh	adegh
bdh	bcfh
acfg	adg
bde	bcef
bdefgh	bcegh
ach	adfh
ace	bcg
bdfg	adef

Plan 9.8.4.   (Continued)

<u>Blocks</u>

<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abf	abgh	cdgh	cdf
cdeg	cdefh	abefh	abeg
abefghj	abej	cdej	cdefghj
cdhj	cdfgj	abfgj	abhj

<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>
adegh	adef	bcef	bcegh
bcfh	bcg	adg	adfh
adj	adfghj	bcfghj	bcj
bcefgj	bcehj	adehj	adefgj

Plan 9.8.8. 9 factors 1/8 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ, FGJ, EFH, AFHJ, AEFG, AGH, AEJ

All main effects and all two-factor interactions are estimable with the exception of:

AB, AC, AD, BC, BD, CD

Blocks

1	2	3	4	5	6	7	8
(1)	abcd	bdc	ace	abf	cdf	adef	bcef
abcdefg	efg	acfg	bdfg	cdeg	abeg	bcg	adg
fgh	abcdfgh	bdefgh	acefgh	abgh	cdgh	adegh	bcegh
abcdeh	eh	ach	bdh	cdefh	abefh	bcfh	adfh
efj	abcdefj	bdfj	acfj	abej	cdej	adj	bcj
abcdgj	gj	acegj	bdegj	cdfgj	abfgj	bcefgj	adefgj
eghj	abcdeghj	bdghj	acghj	abefghj	cdefghj	adfghj	bcfghj
abcdfhj	fhj	acefhj	bdefhj	cdhj	abhj	bcehj	adehj

<u>Plan 9.8.16</u>. 9 factors, 1/8 replication, 4 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J
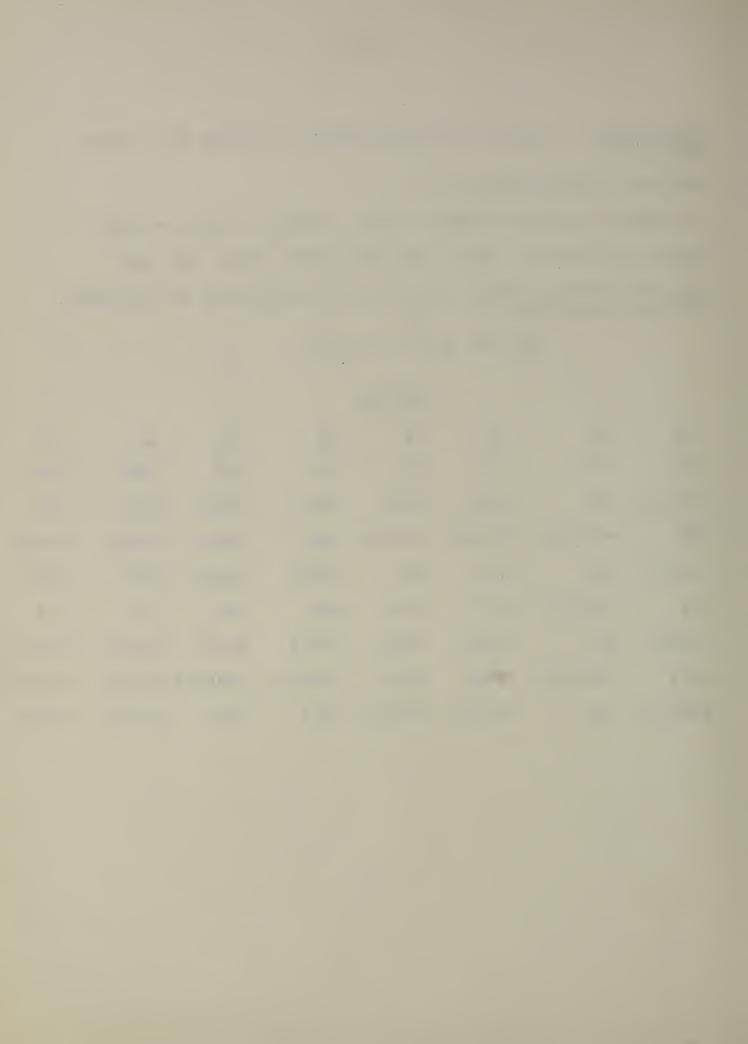
I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ, FGJ, EFH

All main effects and all two-factor interactions are estimable with the <u>exception</u> of:

$$AB, \quad AC, \quad AD, \quad BC, \quad BD, \quad CD .$$

<div align="center">

<u>Blocks</u>

</div>

Combine blocks in Plan 9.8.8 as follows:

<div align="center">

1 and 2

3 and 4

5 and 6

7 and 8 .

</div>

Plan 9.8.32. 9 factors, 1/8 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABEGHJ = ACFGJ = BCEFH = ABCD = CDEGHJ = BDFGJ = ADEFH

Block confounding: EGHJ

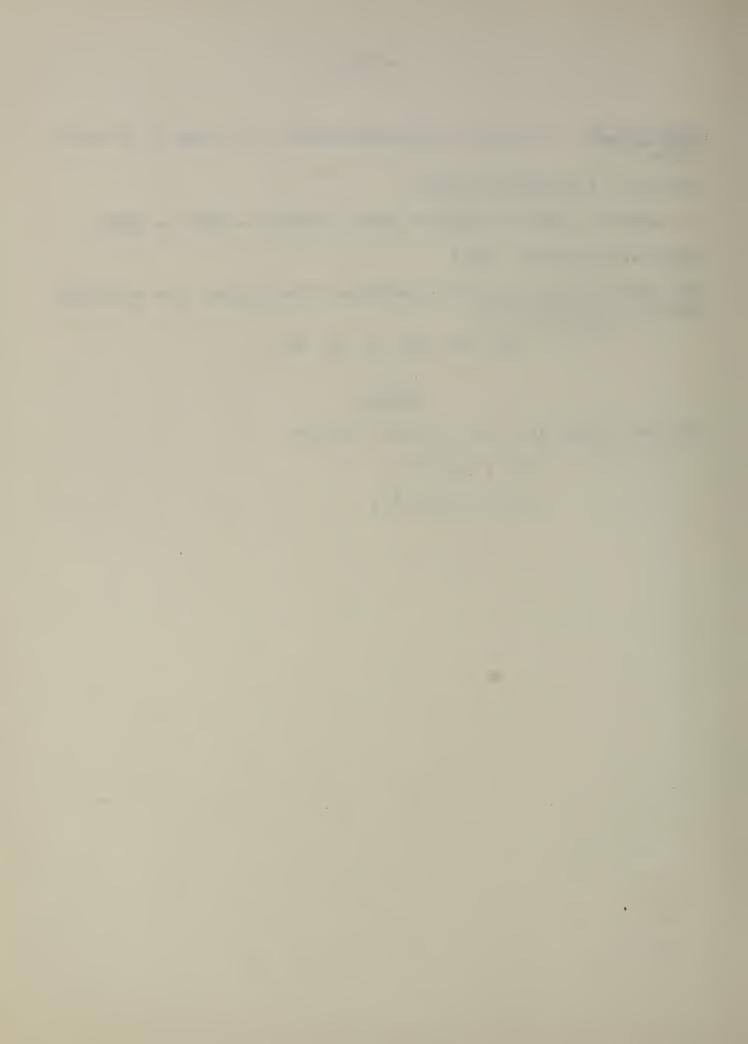All main effects and all two-factor interactions are estimable with the <u>exception</u> of:

$$\text{AB, AC, AD, BC, BD, CD .}$$

<u>Blocks</u>

Combine blocks in Plan 9.8.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 10.8.8. 10 factors, 1/8 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,**H**,J,K

I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK, FHJK, FGK, EFJ, EFGH, EG, EHJ, GHK, JK, EFGHJK, EFK, FGJ, FH

All main effects and all two-factor interactions are estimable with the exception of:

BD, EG, FH, JK

Blocks

1	2	3	4
(1)	eghj	dfk	defghjk
abcd	abcdeghj	abcfk	abcefghjk
aegjk	ahk	adefgj	adfh
bcdegjk	bcdhk	bcefgj	bcfh
acefgh	acfj	acdeghk	acdjk
bdefgh	bdfj	beghk	bjk
cfhjk	cefgk	cdhj	cdeg
abdfhjk	abdefgk	abhj	abeg

5	6	7	8
dhjk	degk	fhj	efg
abchjk	abcegk	abcdfhj	abcdefg
adegh	adj	aefghk	afjk
bcegh	bcj	bcdefghk	bcdfjk
acdefgjk	acdfhk	acegj	ach
befgjk	bfhk	bdegj	bdh
cdf	cdefghj	ck	ceghjk
abf	abefghj	abdk	abdeghjk

Plan 10.8.8. (Continued).

Blocks

9	10	11	12
abej	abgh	abdefjk	abdfghk
cdej	cdgh	cefjk	cfghk
bgk	behjk	bdfg	bdefhj
acdgk	acdehjk	acfg	acefhj
bcfghj	bcef	bcdghjk	bcdek
adfghj	adef	aghjk	aek
abcefhk	abcfgjk	abcdeh	abcdgj
defhk	dfgjk	eh	gj

13	14	15	16
abdehk	abdgjk	abefh	abfgj
cehk	cgjk	cdefh	cdfgj
bdghj	bde	bfghjk	befk
acghj	ace	acdfghjk	acdefk
bcdfgk	bcdefhjk	bcg	bcehj
afgk	aefhjk	adg	adehj
abcdefj	abcdfgh	abcejk	abcghk
efj	fgh	dejk	dghk

Plan 10.8.16.   10 factors, 1/8 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

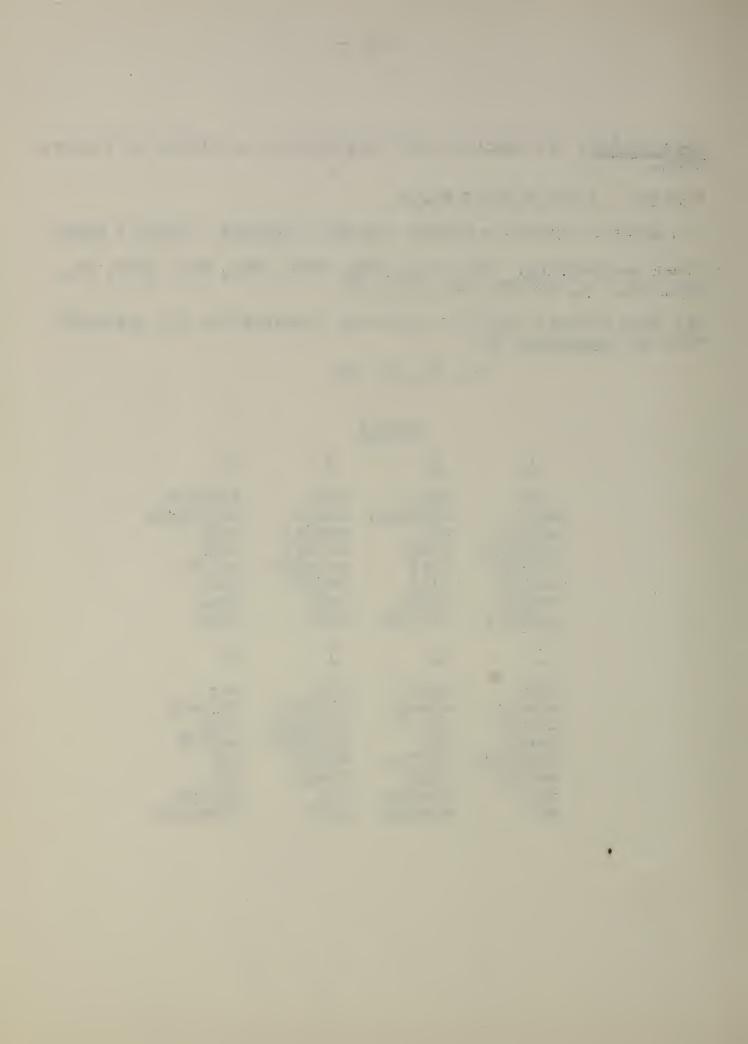I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK, FHJK, FGK, EFJ, EFGH

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 10.8.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16

Plan 10.8.32. 10 factors, 1/8 replication, 4 blocks of 32
units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABEGHJ = ACFGJK = BCEFHK = ABCDK = CDEGHJK = BDFGJ = ADEFH

Block confounding: GHJ, EHK, EGJK

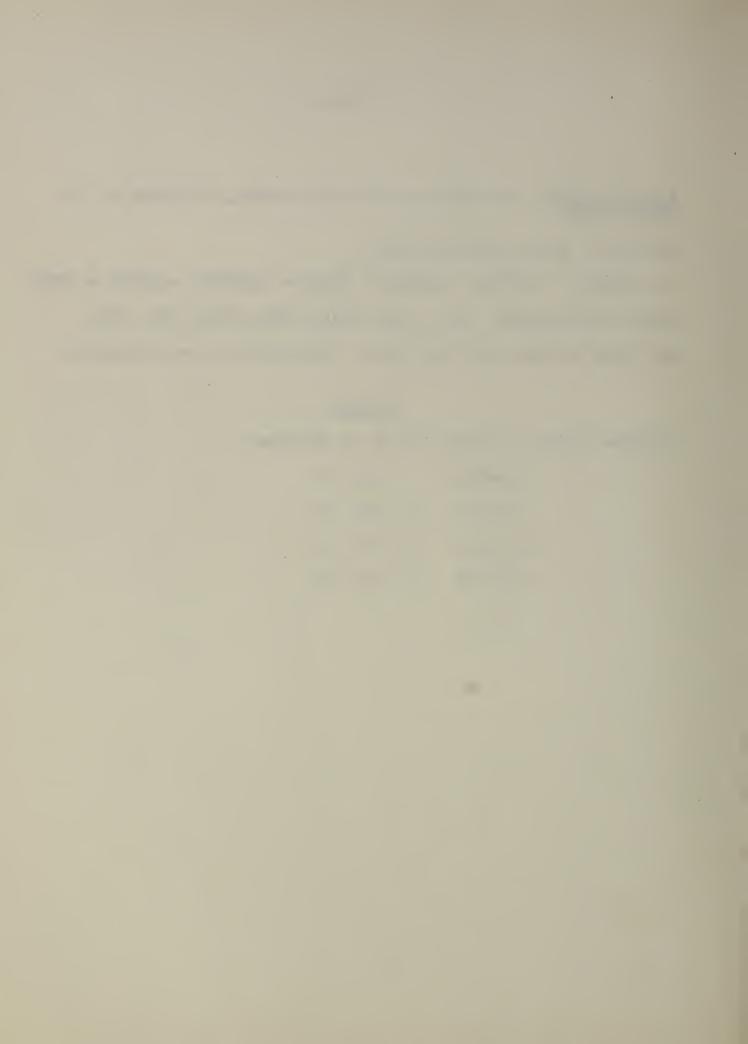All main effects and two factor interactions are estimable.

<div align="center">Blocks</div>

Combine blocks in Plan 10.8.8 as follows:

<div align="center">

1, 2, 3, and 4

5, 6, 7, and 8

9, 10, 11, and 12

13, 14, 15, and 16 .

</div>

Plan 11.8.8. 11 factors, 1/8 replication, 32 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABEGHJ = ACFGJK = BCEFHK = ABCDKL = CDEGHJKL = BDFGJL

   = ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH,

                      EGL , EHJ, GHK, JKL, EFGHJKL, EFK, FGJ,

                      FHL, AEG, AEHJL, AGHKL, AJK, AEFGHJK,

                      AEFKL, AFGJL, AFH, AL, AGHJ, AEHK, AEGJKL,

                      AFHJKL, AFGK, AEFJ, AEFGHL

All main effects and all two-factor interactions are estimable with the exception of:

$$AL, \quad CJ, \quad DG .$$

Blocks

1	2	3	4	5	6	7	8
(1)	eghj	dfk	defghjk	dhjk	degk	fhj	efg
bcdegjk							
bdefgh							
cfhjk							
adfgkl							
abcefjl							
abehkl							
acdghjl							

Plan 11.8.8. (Continued).

Blocks

9	10	11	12
abej	abgh	abdefjk	abdfghk

13	14	15	16
abdehk	abdgjk	abefh	abfgj

17	18	19	20
abdfl	abdefghjl	abkl	abeghjkl

21	22	23	24
abfhjkl	abefgkl	abdhjl	abdegl

25	26	27	28
defjl	dfghl	ejkl	ghkl

29	30	31	32
efhkl	fgjkl	dehl	dgjl

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Plan 11.8.16. 11 factors, 1/8 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABEGHJ = ACFGJK = BCEFHK = ABCDKL = CDEGHJKL = BDFGJL

  = ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH,

EGL, EHJ, GHK, JKL, EFGHJKL, EFK, FGJ, FHL

All main effects and two-factor interactions are estimable.

Blocks

1	2	3	4
(1)	eghj	dfk	defghjk
bcdegjk			
bdefgh	**5**	**6**	**7**
cfhjk	dhjk	degk	fhj
adfgkl			
abcdfjl	**8**	**9**	**10**
abehkl	efg	abej	abgh
acdghjl			
defjl	**11**	**12**	**13**
bcfgkl	abdefjk	abdfghk	abdehk
bghjl			
cdehkl			
aegjk	**14**	**15**	16
abcd	abdgjk	abefh	abfgj
abdfhjk			
acefgh			

Plan 11.8.32. 11 factors, 1/8 replication, 8 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L
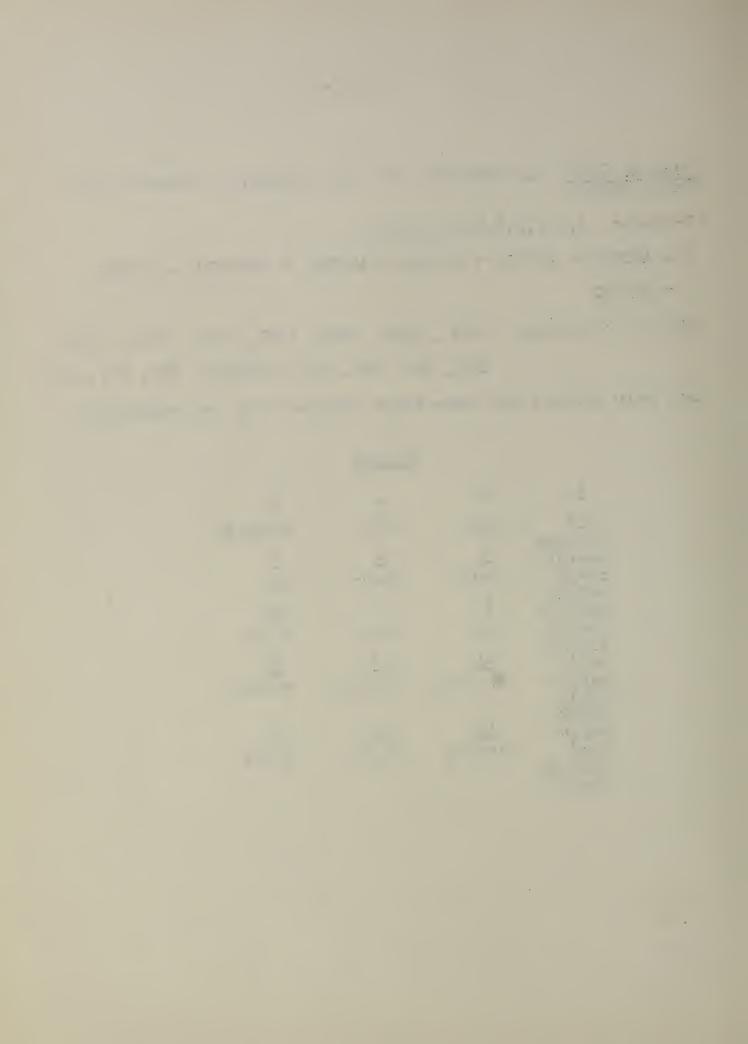
I = ABEGHJ = ACFGJK = BCEFHK = ABCDIL = CDEGHJKL

= BDFGJL = ADEFHL

Block confounding: GHJL, EHKL, EGJK, FHJK, FGKL, EFJL, EFGH

All main effects and two-factor interactions are estimable.

Blocks

1

(1)	abfgj
bcdegjk	acdefk
bdefgh	adehj
cfhjk	abcghk
adfgkl	bdjkl
abcefjl	cegl
abehkl	efghjkl
acdghjl	bcdfhl
defjl	abdegl
bcfgkl	acjkl
bghjl	afhl
cdehkl	abcdefghjkl
aegjk	befk
abcd	cdfgj
abdfhjk	dghk
acefgh	bcehj

2	3	4	5
eghj	dfk	defghjk	dhjk

6	7	8
degk	fhj	efg

Plan 7.16.4. 7 factors, 1/16 replication, 2 blocks of 4 units each.

Factors: A,B,C,D,E,F,G

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFG

  = EFG = CDG = ABG = ADF = BCF = BDE = ACE

Block confounding: A

All main effects <u>except</u> A are estimable. They are identified with two-factor interactions as follows:

B = AG = CF = DE	E = FG = BD = AC
C = DG = BF = AE	F = EG = AD = BC
D = CG = AF = BE	G = EF = CD = AB

Blocks

<u>1</u>	<u>2</u>
(1)	acfg
cdef	adeg
bdfg	abcd
bceg	abef

<u>Plan 7.16.8.</u> 7 factors, 1/16 replication, 1 block of 8 units each.

Factors: A,B,C,D,E,F,G

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFG

  = EFG = CDG = ABG = ADF = BCF = BDE = ACE

All main effects are estimable. They are identified with two-factor interactions as follows:

$$A = BG = DF = CE \qquad\qquad E = FG = BD = AC$$

$$B = AG = CF = DE \qquad\qquad F = EG = AD = BC$$

$$C = DG = BF = AE \qquad\qquad G = EF = CD = AB$$

$$D = CG = AF = BE$$

<u>Block</u>

(1)	abcd
acfg	bdfg
adeg	bceg
cdef	abef

<u>Plan 8.16.4.</u> 8 factors, 1/16 replication, 4 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH

  = EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

Block confounding: AB, AC, BC

All main effects are estimable.

<div align="center">

<u>Blocks</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	acfg	cdef	adeg
abcd	bdfg	abef	bceg
abcdefgh	bdeh	abgh	bcfh
efgh	aceh	cdgh	adfh

</div>

Plan 8.16.8. 8 factors, 1/16 replication, 2 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH

   = EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

Block confounding: AB

All main effects are estimable.

Blocks

1	2
(1)	acfg
cdef	adeg
abcd	bdfg
abef	bceg
abcdefgh	bdeh
abgh	bcfh
efgh	aceh
cdgh	adfh

Plan 8.16.16. 8 factors 1/16 replication, 1 block of 16 units each.
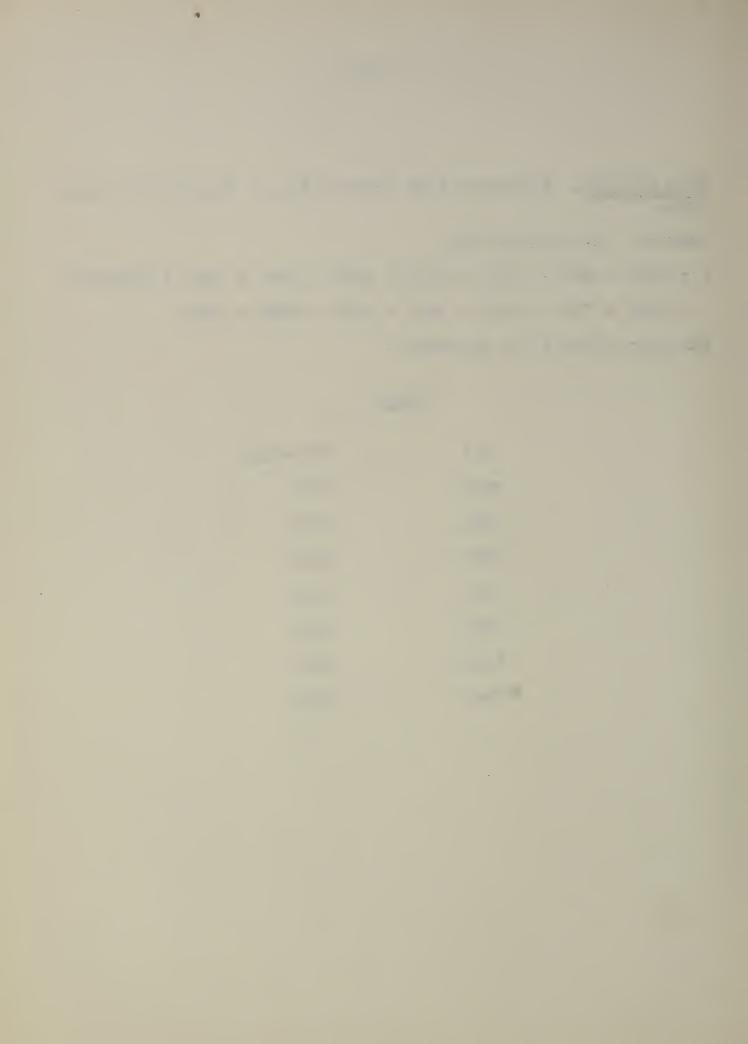
Factors: A,B,C,D,E,F,G,H

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGH

  = EFGH = CDGH = ABGH = ADFH = BCFH = BDEH = ACEH

All main effects are estimable.

Block

(1)	abcdefgh
acfg	bdeh
adeg	bcfh
cdef	abgh
abcd	efgh
bdfg	aceh
bceg	adfh
abef	cdgh

<u>Plan 9.16.4.</u> 9 factors, 1/16 replication, 8 blocks of 4 units
each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ

  = EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB, AC, BC, AE, BE, CE, ABCE

All main effects and the following two-factor interactions are
estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ,
GH,GJ,HJ.

<u>Blocks</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	aceh	cdgh	adeg
abcdefgh	bdfg	abef	bcfh
abcdefgj	bdfghj	abefhj	bcfj
hj	acej	cdgj	adeghj

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
abcd	bdeh	abgh	bceg
efgh	acfg	cdef	adfh
efgj	acfghj	cdefhj	adfj
abcdhj	bdej	abgj	bceghj

Plan 9.16.8. 9 factors, 1/16 replication, 4 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ

= EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB, AC, BC

All main effects and the following two-factor interactions are estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ, GH, GJ, HJ.

Blocks

1	2	3	4
(1)	aceh	cdgh	adeg
abcd	bdeh	abgh	bceg
abcdefgh	bdfg	abef	bcfh
efgh	acfg	cdef	adfh
efgj	acfghj	cdefhj	adfj
abcdefgj	bdfghj	abefhj	bcfj
abcdhj	bdej	abgj	bceghj
hj	acej	cdgj	adeghj

Plan 9.16.16. 9 factors, 1/16 replication, 2 blocks of 16 units each.
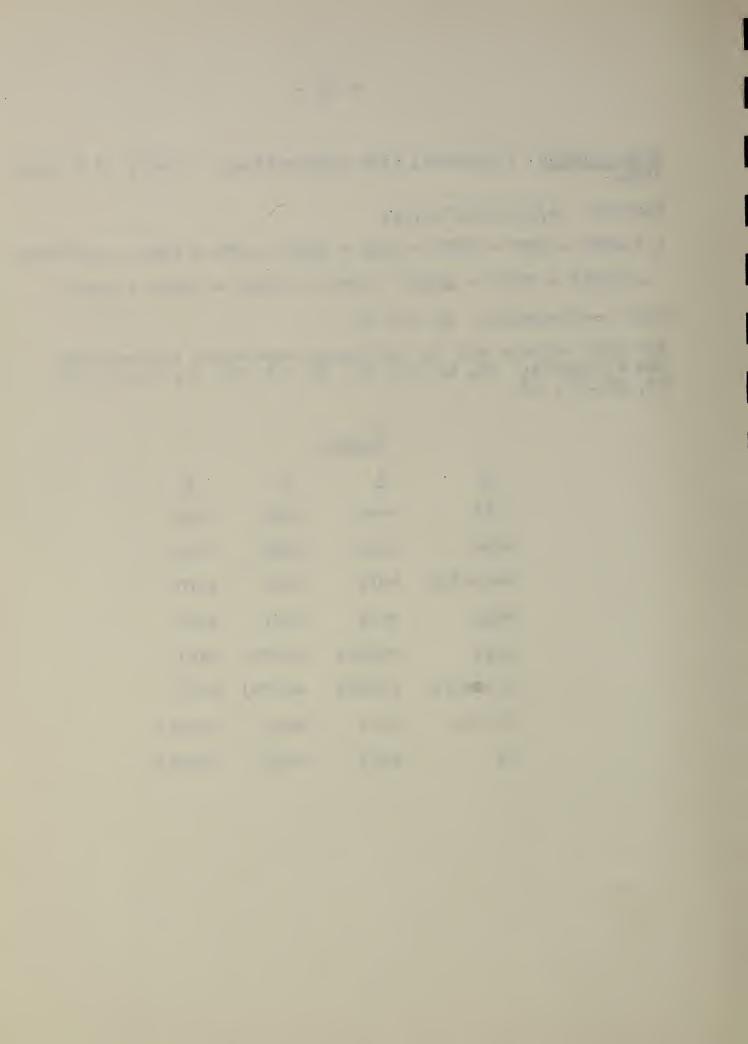
Factors: A,B,C,D,E,F,G,H,J

I = ABCD = ABEF = CDEF = BCEG = ADEG = ACFG = BDFG = ABCDEFGHJ

  = EFGHJ = CDGHJ = ABGHJ = ADFHJ = BCFHJ = BDEHJ = ACEHJ

Block confounding: AB

All main effects and the following two-factor interactions are estimable: AH, AJ, BH, BJ, CH, CJ, DH, DJ, EH, EJ, FH, FJ, GH, GJ, HJ.

Blocks

1		2	
(1)	cdgh	aceh	adeg
abcd	abgh	bdeh	bceg
abcdefgh	abef	bdfg	bcfh
efgh	cdef	acfg	adfh
efgj	cdefhj	acfghj	adfj
abcdefgj	abefhj	bdfghj	bcfj
abcdhj	abgj	bdej	bceghj
hj	cdgj	acej	adeghj

Plan 10.16.8. 10 factors, 1/16 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K

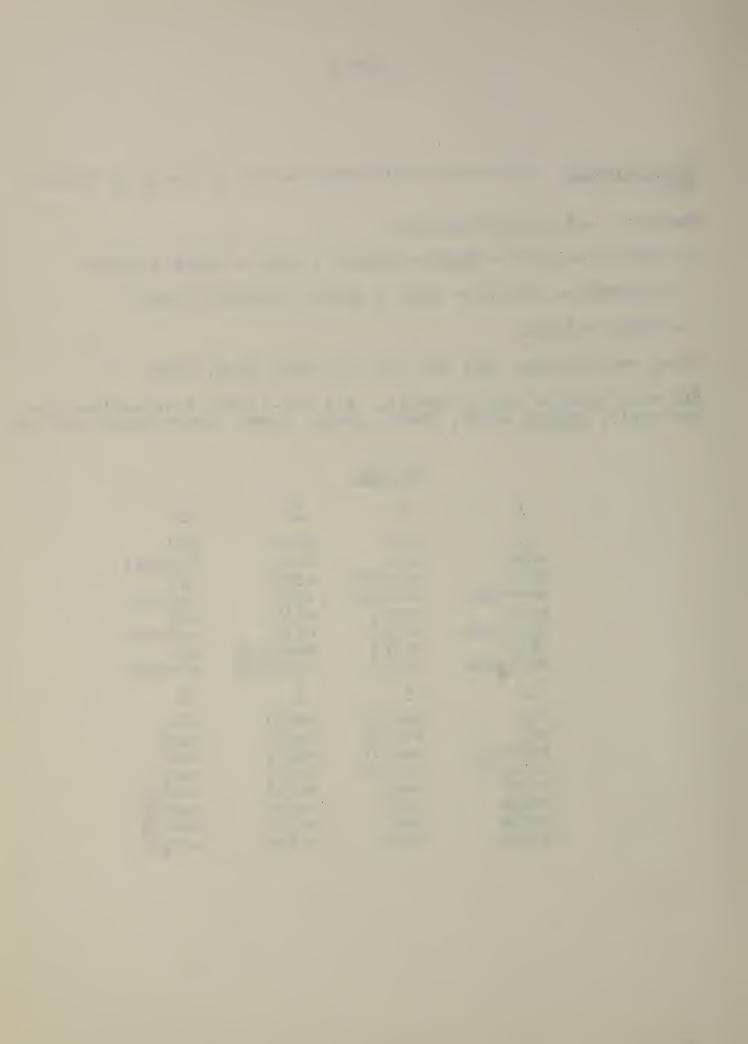I = ABCDJK = ABEFJ = CDEFK = BCEGJK = ADEG = ACFGK = BDFGJ

  = ABCDEFGH = EFGHJK = CDGHJ = ABGHK = ADFHJK = BCFH

  = BDEHK = ACEHJ

Block confounding: AD, AE, DE, BC, ABCD, ABCE, BCDE

All main effects are estimable. All two-factor interactions are estimable, except AD=EG, AE=DG, AG=DE, BC=FH, BF=CH, BH=CF, and JK.

Blocks

1	2	3	4
(1)	aek	abcd	bhj
adeg	dgk	bceg	abdeghj
bcfh	abcefhk	adfh	cfj
abcdefgh	bcdfghk	efgh	acdefgj
bcjk	abcej	adjk	chk
abcdegjk	bcdgj	egjk	acdeghk
fhjk	aefhj	abcdfhjk	bfk
adefghjk	dfghj	bcefghjk	abdefgk

5	6	7	8
dej	abehjk	acdhj	bdeh
agj	bdghjk	ceghj	abgh
bcdefhj	acefjk	abdfj	cdef
abcfghj	cdfgjk	befgj	acfg
bcdek	aceh	abdhk	cdehjk
abcgk	cdgh	beghk	acghjk
defhk	abef	acdfk	bdefjk
afghk	bdfg	cefgk	abfgjk

Plan 10.16.16. 10 factors, 1/16 replication, 4 blocks of 16 units each.

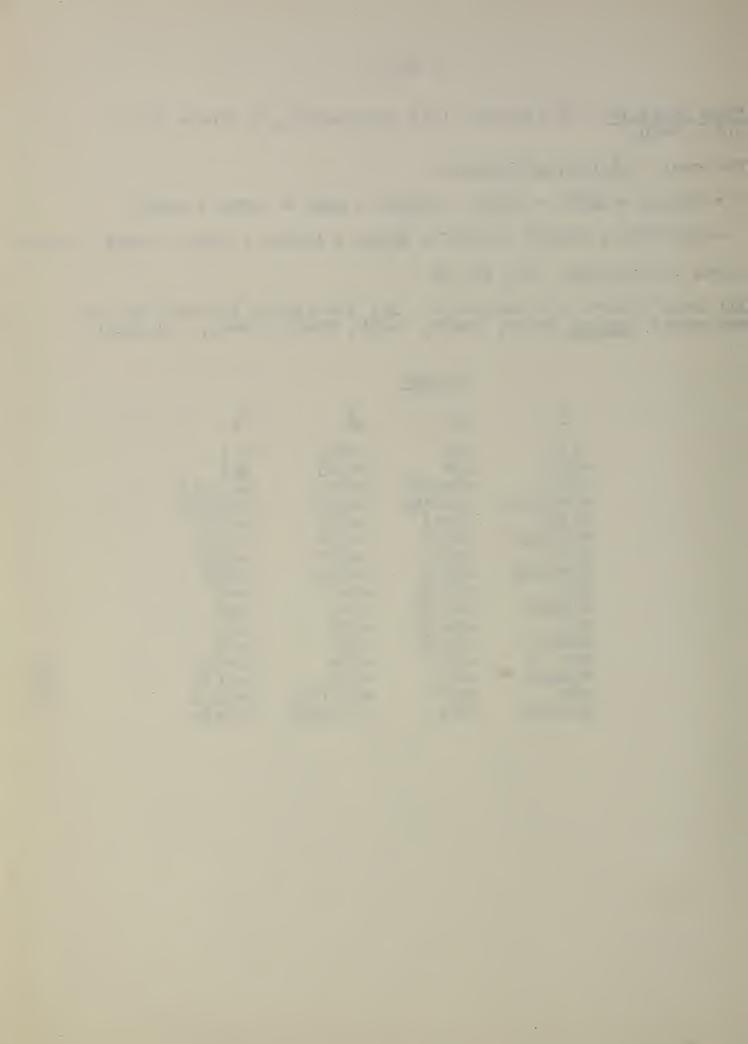Factors: A,B,C,D,E,F,G,H,J,K

I = ABCDJK = ABEFJ = CDEFK = BCEGJK = ADEG = ACFGK = BDFGJ

  = ABCDEFGH = EFGHJK = CDGHJ = ABGHK = ADFHJK = BCFH = BDEHK = ACEHJ

Block confounding: AD, AE, DE

All main effects are estimable. All two-factor interactions are estimable, **except** AD=EG, AE=DG, AG=DE, BC=FH, BF=CH, and BH=CF.

Blocks

1	2	3	4
(1)	aek	acdhj	dej
adeg	dgk	ceghj	agj
bcfh	abcefhk	abdfj	bcdefhj
abcdefgh	bcdfghk	befgj	abcfghj
bcjk	abcej	abdhk	bcdek
abcdegjk	bcdgj	beghk	abcgk
fhjk	aefhj	acdfk	defhk
adefghjk	dfghj	cefgk	afghk
bhj	abehjk	abcd	bdeh
abdeghj	bdghjk	bceg	abgh
cfj	acefjk	adfh	cdef
acdefgj	cdfgjk	efgh	acfg
chk	aceh	adjk	cdehjk
acdeghk	cdgh	egjk	acghjk
bfk	abef	abcdfhjk	bdefjk
abdefgk	bdfg	bcefghjk	abfgjk

Plan 11.16.8. 11 factors, 1/16 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKL = ADEGL = ACFGK = BDFGJ

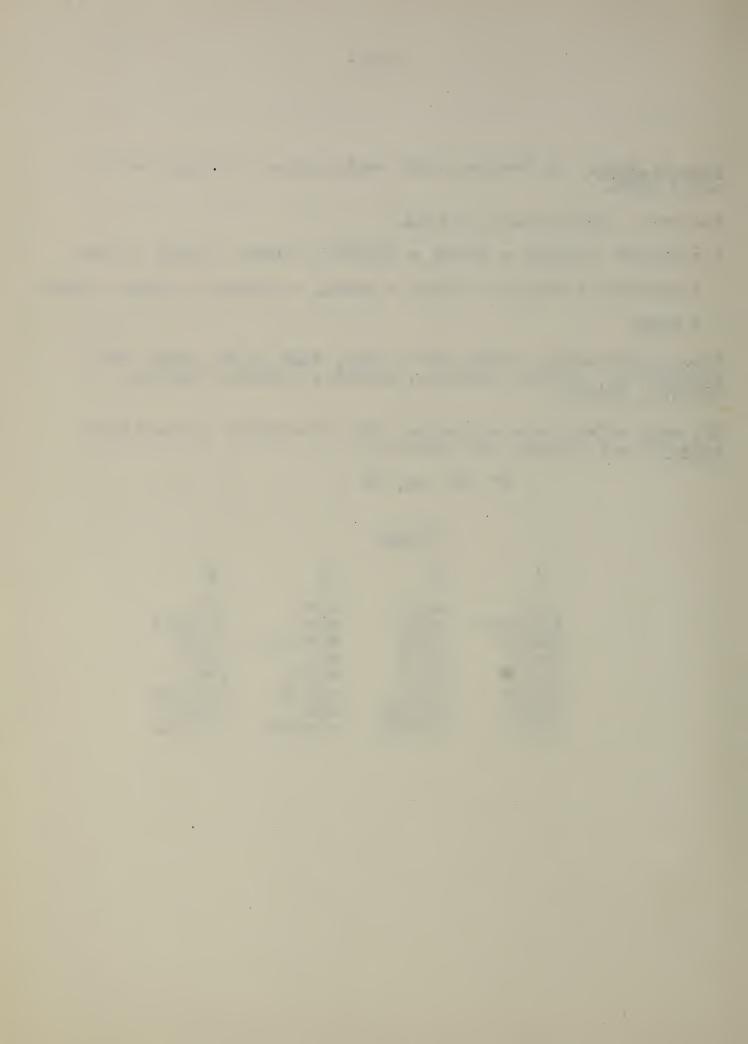   = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKL = BCFHL = BDEHK

   = ACEHJ

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL, CDFHJKL, CEGHJKL.

All main effects are estimable. All two-factor interactions except the following are estimable:

EF, DG, AC, BH .

Blocks

1	2	3	4
(1)	abdfj	abcd	cfj
abcdefgh	ceghj	efgh	abdeghj
defgjl	abegl	abcefgjl	cdegl
abchjl	cdfhl	dhjl	abfhl
acefjk	bcdek	bdefjk	aek
bdghjk	afghk	acghjk	bcdfghk
acdgkl	bcfgjkl	bgkl	adfgjkl
befhkl	adehjkl	acdefhkl	bcehjkl

Plan 11.16.8. (Continued).

Blocks

5	6	7	8
bceg	bhj	abef	dej
adfh	acdefgj	cdgh	abcfghj
bcdfjl	bdefghl	abdgjl	fgl
aeghjl	acl	cefhjl	abcdehl
abfgjk	abcefhk	bcjk	acdfk
cdehjk	dgk	adefghjk	beghk
abdekl	abcdghjkl	bcdefgkl	acegljkl
cfghkl	efjkl	ahkl	bdfhjkl

9	10	11	12
adeg	acdhj	cdef	abcej
bcfh	befgj	abgh	dfghj
afjl	acefghl	cgjl	abcdfgl
bcdeghjl	bdl	abdefhjl	ehl
cdfgjk	defhk	adjk	bfk
abehjk	abcgk	bcefghjk	acdeghk
cekl	ghjkl	aefgkl	bdegjkl
abdfghkl	abcdefjkl	bcdhkl	acfhjkl

13	14	15	16
acfg	bcdgj	bdfg	agj
bdeh	aefhj	aceh	bcdefhj
acdejl	bcefl	bejl	adefl
bfghjl	adghl	acdfghjl	bcghl
egjk	abdefgk	abcdegjk	cefgk
abcdfhjk	chk	fhjk	abdhk
dfkl	abjkl	abcfkl	cdjkl
abceghkl	cdefghjkl	deghkl	abefghjkl

Plan 11.16.16. 11 factors, 1/16 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKL = ADEGL = ACFGK = BDFGJ

  = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKL = BCFHL

  = BDEHK = ACEHJ

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 11.16.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16 .

<u>Plan 12.16.8.</u> 12 factors, 1/16 replication, 32 blocks of 8
units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKLM = ADEGLM = ACFGKM

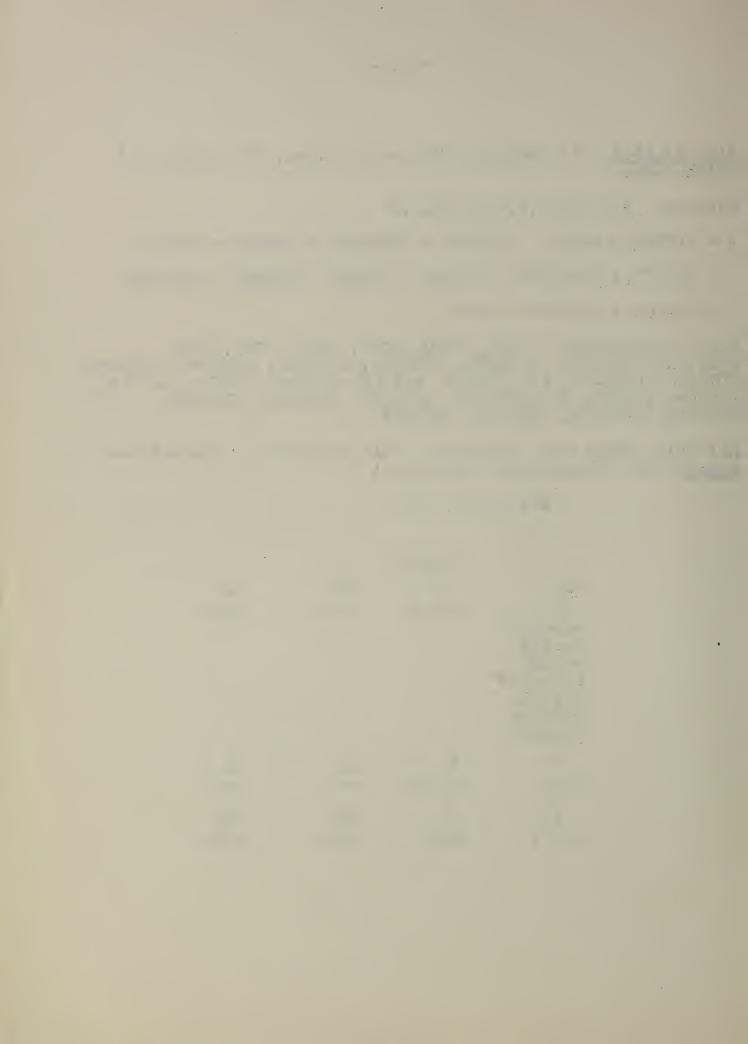 = BDFGJM = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKLM

  = BCFHLM = BDEHKM = ACEHJM

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG,
ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL,
CDFHJKL, CEGHJKL, LM, DEFGLM, BCFGLM, BCDELM, ACEFLM, ACDGLM,
ABEGLM, ABDFLM, ABCDEFGHJKM, ABCHJKM, ADEHJKM, AFGHJKM,
BDGHJKM, BEFHJKM, CDFHJKM, CEGHJKM

All main effects are estimable. All two factor interactions
<u>except</u> the following are estimable:

AF, GH, CE, BD .

<u>Blocks</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	defgjl	abcd	bgkl
abcdefgh			
acefjk			
bdghjk			
bcdejlm			
afghjlm			
abdfklm			
ceghklm			

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bceg	bcdfjl	abef	ahkl

<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>
abdfj	abegl	adeg	afjl

Plan 12.16.8. (Continued).

Blocks

13	14	15	16
cdef	cgjl	cfj	cdegl
17	18	19	20
acfg	dfkl	bhj	acl
21	22	23	24
dej	fgl	bdfg	bejl
25	26	27	28
acdhj	bdl	bfk	ehl
29	30	31	32
chk	bcefl	agj	adefl

Plan 12.16.16. 12 factors, 1/16 replication, 16 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCDJK = ABEFJL = CDEFKL = BCEGJKLM = ADEGLM = ACFGKM

  = BDFGJM = ABCDEFGH = EFGHJK = CDGHJL = ABGHKL = ADFHJKLM
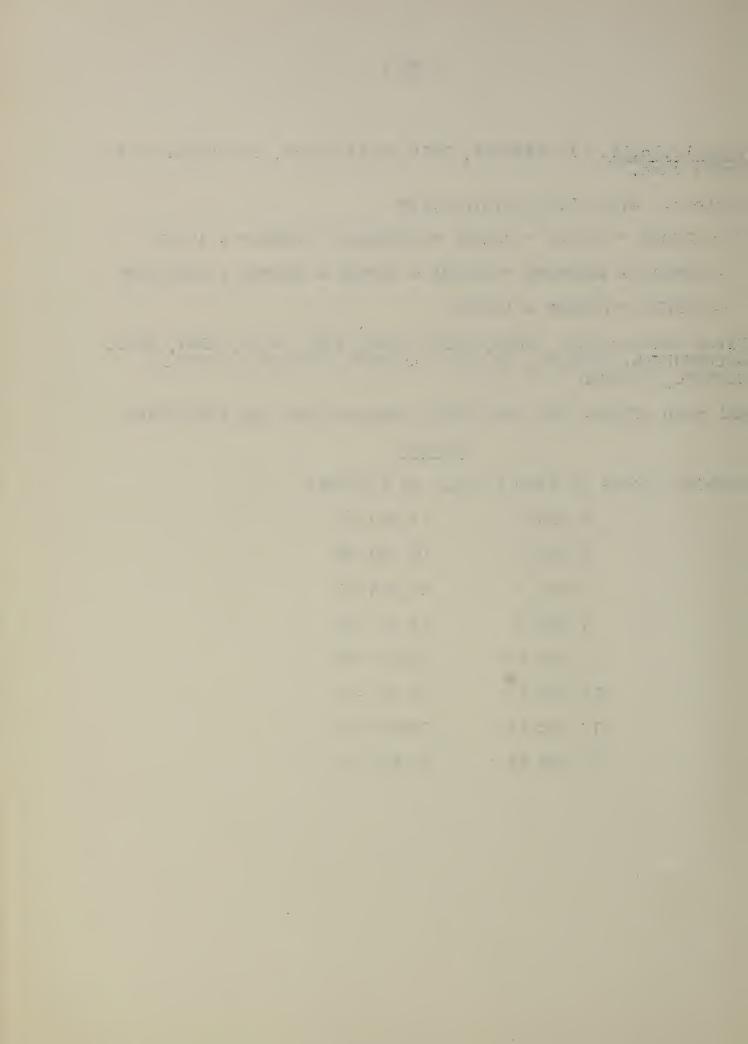
  = BCFHLM = BDEHKM = ACEHJM

Block confounding: DEFG, BCFG, BCDE, ACEF, ACDG, ABEG, ABDF, ABCDEFGHJKL, ABCHJKL, ADEHJKL, AFGHJKL, BDGHJKL, BEFHJKL, CDFHJKL, CEGHJKL

All main effects and two-factor interactions are estimable.

Blocks

Combine blocks in Plan 12.16.8 as follows:

     1 and 2   17 and 18

     3 and 4   19 and 20

     5 and 6   21 and 22

     7 and 8   23 and 24

     9 and 10   25 and 26

    11 and 12   27 and 28

    13 and 14   29 and 30

    15 and 16   31 and 32 .

Plan 10.32.4. 10 factors, 1/32 replication, 8 blocks of 4 units each.

Factors: A,B,C,D,E,F,G,H,J,K

I = ABCD = ABEF = CDEF = ABGH = CDGH = EFGH = ABCDEFGH = ABJK

  = CDJK = EFJK = ABCDEFJK = GHJK = ABCDGHJK = ABEFGHJK = CDEFGHJK

  = ACEGJ = BDEGJ = BCFGJ = ADFGJ = BCEHJ = ADEHJ = ACFHJ = BDFHJ

  = BCEGK = ADEGK = ACFGK = BDFGK = ACEHK = BDEHK = BCFHK = ADFHK

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

All main effects are estimable, but no two-factor interactions are estimable.

<center>Blocks</center>

1	2	3	4
(1)	cdef	abcd	abef
abcdefgh	abgh	efgh	cdgh
ghjk	cdefghjk	abcdghjk	abefghjk
abcdefjk	abjk	efjk	cdjk

5	6	7	8
bcegj	bdfgj	adegj	acfgj
adfhj	acehj	bcfhj	bdehj
bcehk	bdfhk	adehk	acfhk
adfgk	acegk	bcfgk	bdegk

Plan 10.32.8. 10 factors, 1/32 replication, 4 blocks of 8
units each.

Factors: A,B,C,D,E,F,G,H,J,K

I: Same as Plan 10.32.4

Block confounding: CD, CE, DE

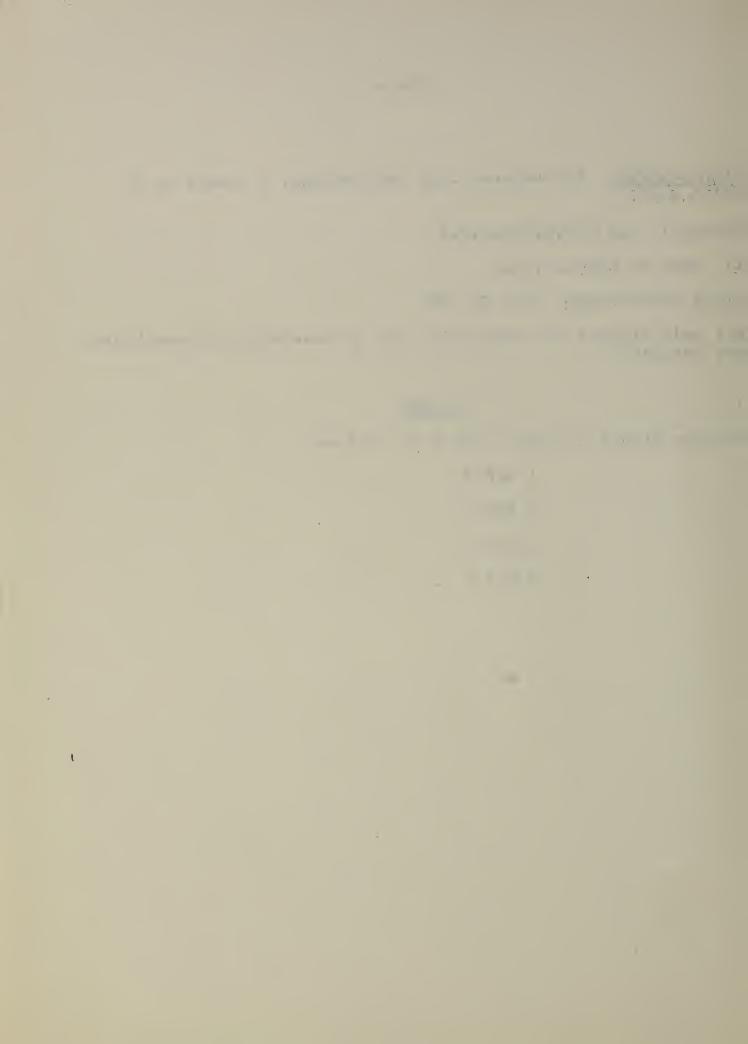All main effects are estimable, but no two-factor interactions
are estimable.

Blocks

Combine blocks in Plan 10.32.4 as follows:

> 1 and 2
>
> 3 and 4
>
> 5 and 6
>
> 7 and 8 .

Plan 10.32.16. 10 factors, 1/32 replication, 2 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K

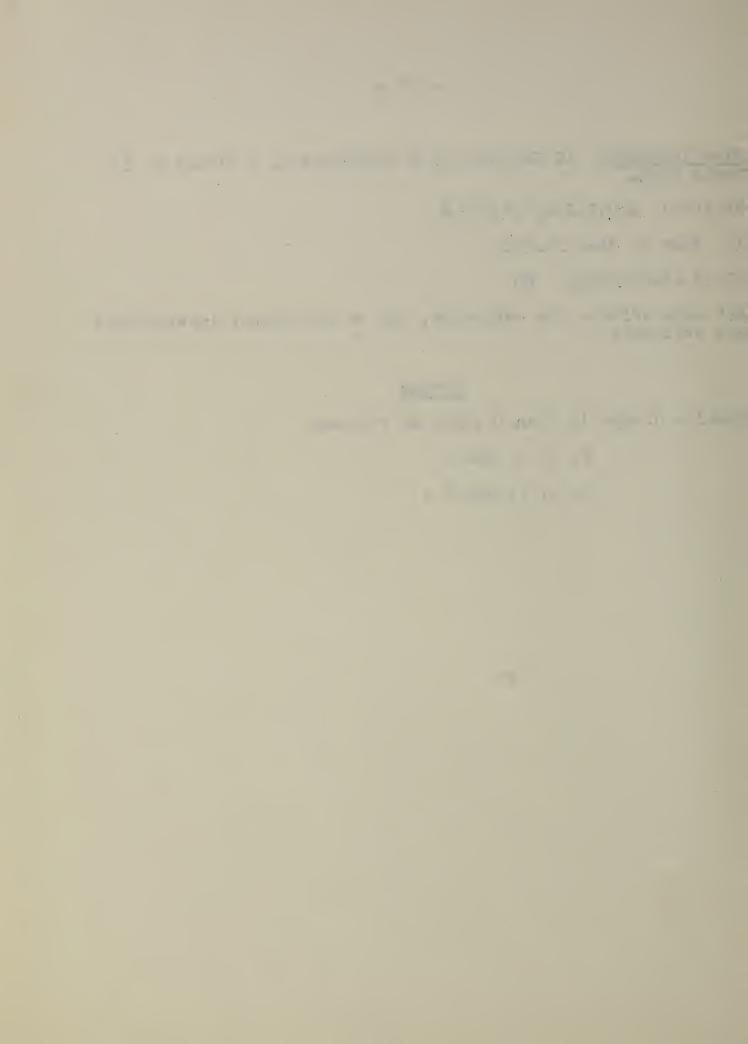I: Same as Plan 10.32.4

Block confounding: CD

All main effects are estimable, but no two-factor interactions are estimable.

Blocks

Combine blocks in Plan 10.32.4 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 11.32.8. 11 factors, 1/32 replication, 8 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I = ABCDL = ABEFL = CDEF = ABGH = CDGHL = EFGHL = ABCDEFGH

  = ABJK = CDJKL = EFJKL = ABCDEFJK = GHJK = ABCDGHJKL

  = ABEFGHJKL = CDEFGHJK = ACEGJL = BDEGJ = BCFGJ = ADFGJL

  = BCEHJL = ADEHJ = ACFHJ = BDFHJL = BCEGKL = ADEGK = ACFGK

  = BDFGKL = ACEHKL = BDEHK = BCFHK = ADFHKL

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

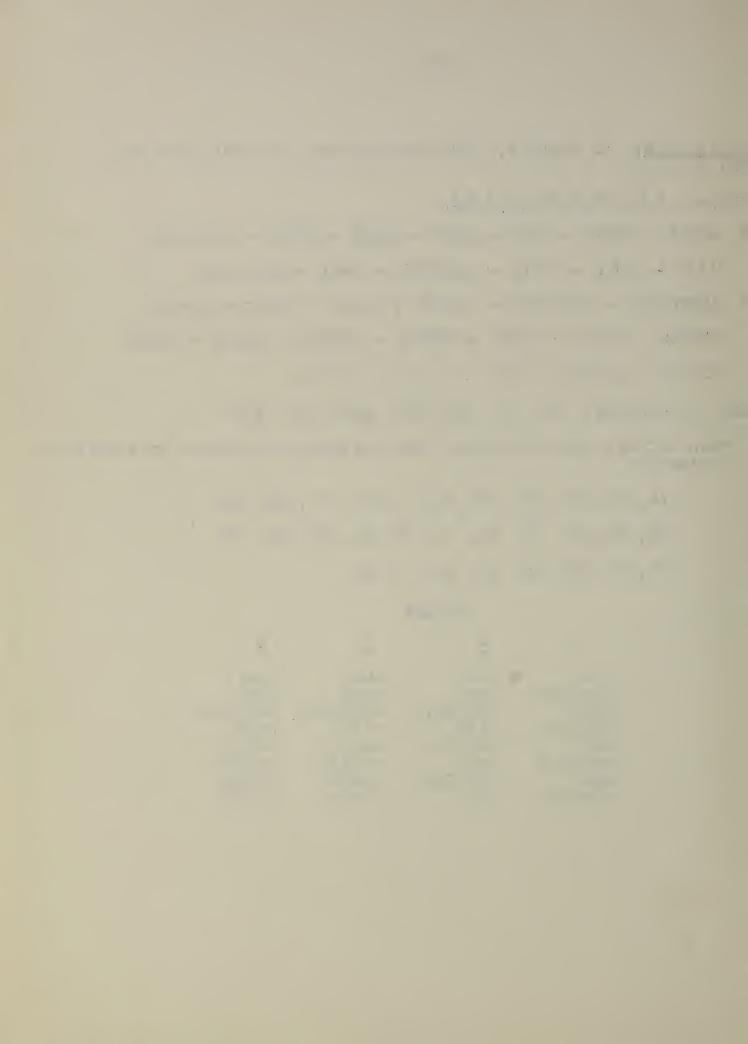All main effects are estimable; the following two-factor interactions are estimable:

  AL, BC, BD, BE, BF, BL, CG, CH, CJ, CK, CL,

  DG, DH, DJ, DK, DL, EG, EH, EJ, EK, EL, FG,

  FH, FJ, FK, FL, GL, HL, JL, KL

<u>Blocks</u>

<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	cdef	abcd	abef
abcdefgh	abgh	efgh	cdgh
ghjk	cdefghjk	abcdghjk	abefghjk
abcdefjk	abjk	efjk	cdjk
bhjl	bcdefhjl	acdhjl	aefhjl
acdefgjl	agjl	befgjl	bcdgjl
bgkl	bcdefgkl	acdgkl	aefgkl
acdefhkl	ahkl	befhkl	bcdhkl

Plan 11.32.8. (Continued).

Blocks

<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
bcegj	bdfgj	adegj	acfgj
adfhj	acehj	bcfhj	bdehj
bcehk	bdfhk	adehk	acfhk
adfgk	acegk	bcfgk	bdegk
ceghl	dfghl	abdeghl	abcfghl
abdfl	abcel	cfl	del
cejkl	dfjkl	abdejkl	abcfjkl
abdfghjkl	abceghjkl	cfghjkl	deghjkl

Plan 11.32.16.   11 factors, 1/32 replication, 4 blocks of 16 units each.

Factors: **A**,B,C,D,E,F,G,H,J,K,L

I: Same as Plan 11.32.8

Block confounding: CD, CE, DE

All main effects and the following two-factor interactions are estimable:

> AC, AD, AE, AF, AL, BC, BD, BE, BF, BL, CG,
>
> CH, CJ, CK, CL, DG, DH, DJ, DK, DL, EG, EH,
>
> EJ, EK, EL, FG, FH, FJ, FK, FL, GL, HL, JL,
>
> KL

Blocks

Combine blocks in Plan 11.32.8 as follows:

> 1 and 2
>
> 3 and 4
>
> 5 and 6
>
> 7 and 8   .

Plan 11.32.32. 11 factors, 1/32 replication, 2 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L

I: Same as Plan 11.32.8

Block confounding: CD

All main effects and the following two-factor interactions are estimable:

AC, AD, AE, AF, AL, BC, BD, BE, BF, BL, CG,

CH, CJ, CK, CL, DG, DH, DJ, DK, DL, EG, EH,
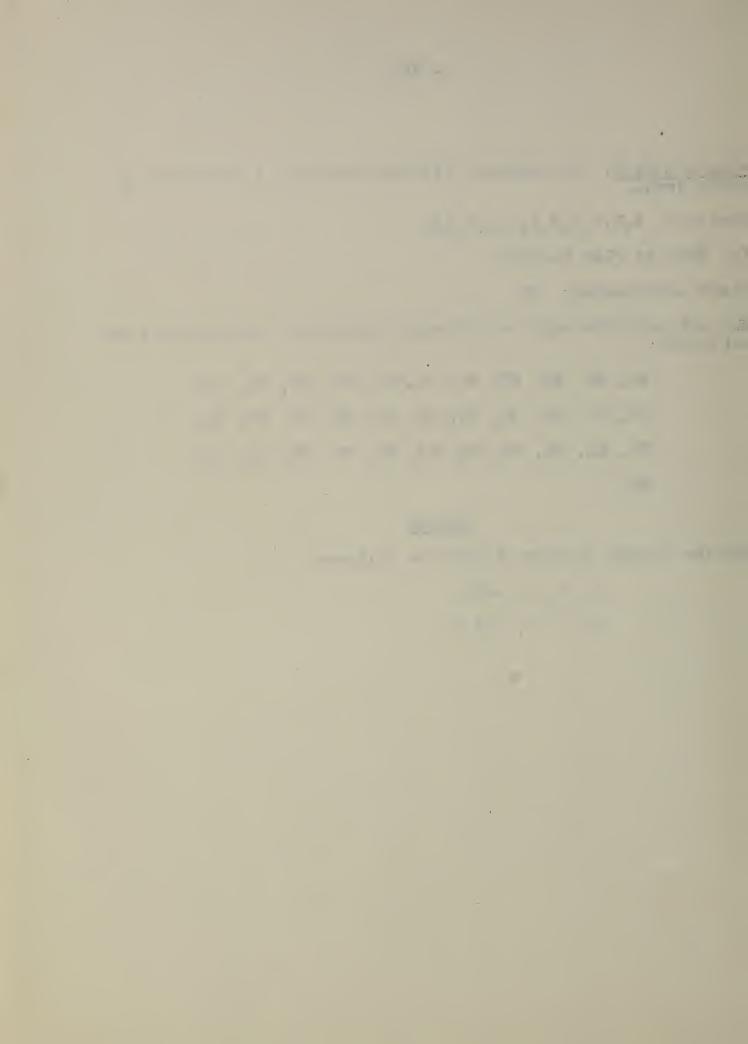
EJ, EK, EL, FG, FH, FJ, FK, FL, GL, HL, JL,

KL

Blocks

Combine blocks in Plan 11.32.8 as follows:

1, 2, 3, and 4

5, 6, 7, and 8 .

Plan 12.32.8. 12 factors, 1/32 replication, 16 blocks of 8 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I = ABCDLM = ABEFL = CDEFM = ABGHM = CDGHL = EFGHLM = ABCDEFGH

 = ABJKM = CDJKL = EFJKLM = ABCDEFJK = GHJK = ABCDGHJKLM

 = ABEFGHJKL = CDEFGHJKM = ACEGJL = BDEGJM = BCFGJ = ADFGJLM

 = BCEHJLM = ADEHJ = ACFHJM = BDFHJL = BCEGKLM = ADEGK = ACFGKM
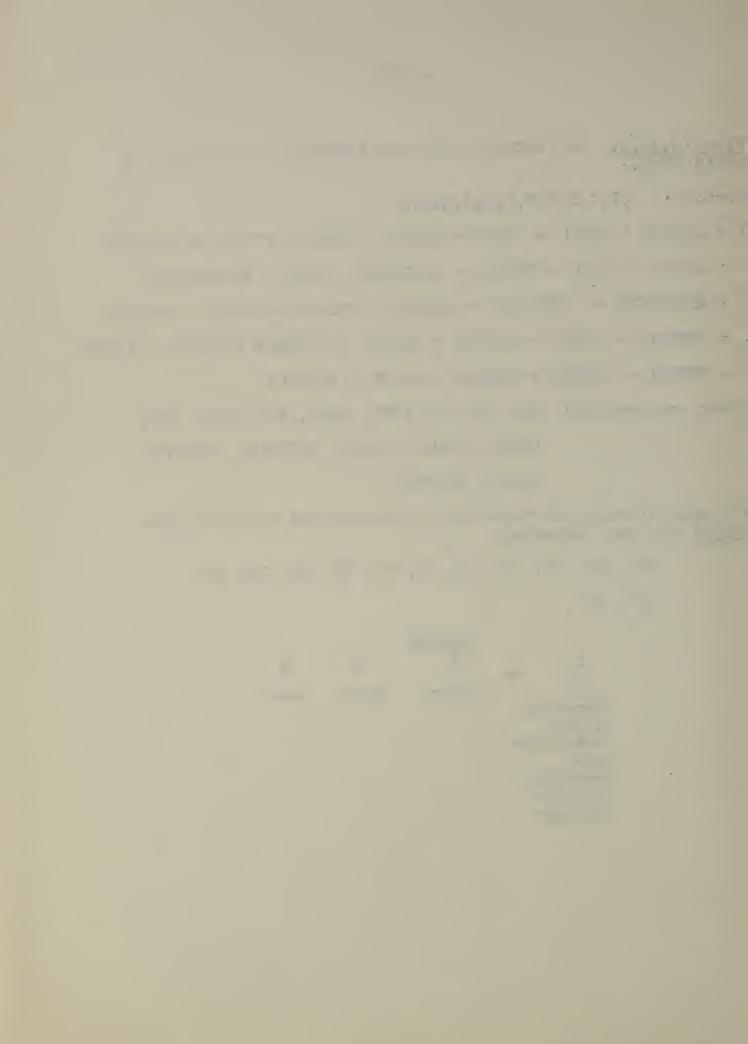
 = BDFGKL = ACEHKL = BDEHKM = BCFHK = ADFHKLM

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF, KLM,

CDKLM, CEKLM, DEKLM, ACEFKLM, ADEFKLM,

AFKLM, ACDFKLM

All main effects and two-factor interactions are estimable <u>except</u> for the following:

AF, BG, CD, CE, CH, DE, DH, EH, GH, GJ, HJ,

JK, JM .

Blocks			
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
(1)	bcdem	abcd	aem
abcdefgh			
afjkm			
bcdeghjkm			
bgkl			
acdefhkl			
abfgjlm			
cdehjlm			

Plan 12.32.8. (Continued).

Blocks

5	6	7	8
cdef	bfm	abef	acdfm

9	10	11	12
del	bclm	abcel	adlm

13	14	15	16
cfl	bdeflm	abdfl	aceflm

Plan 12.32.16. 12 factors, 1/32 replication, 8 blocks of 16 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I: Same as Plan 12.32.8

Block confounding: CD, CE, DE, ACEF, ADEF, AF, ACDF

All main effects and two-factor interactions are estimable <u>except</u> for the following:

    AF, CD, CE, DE, GH, GJ, HJ, JK
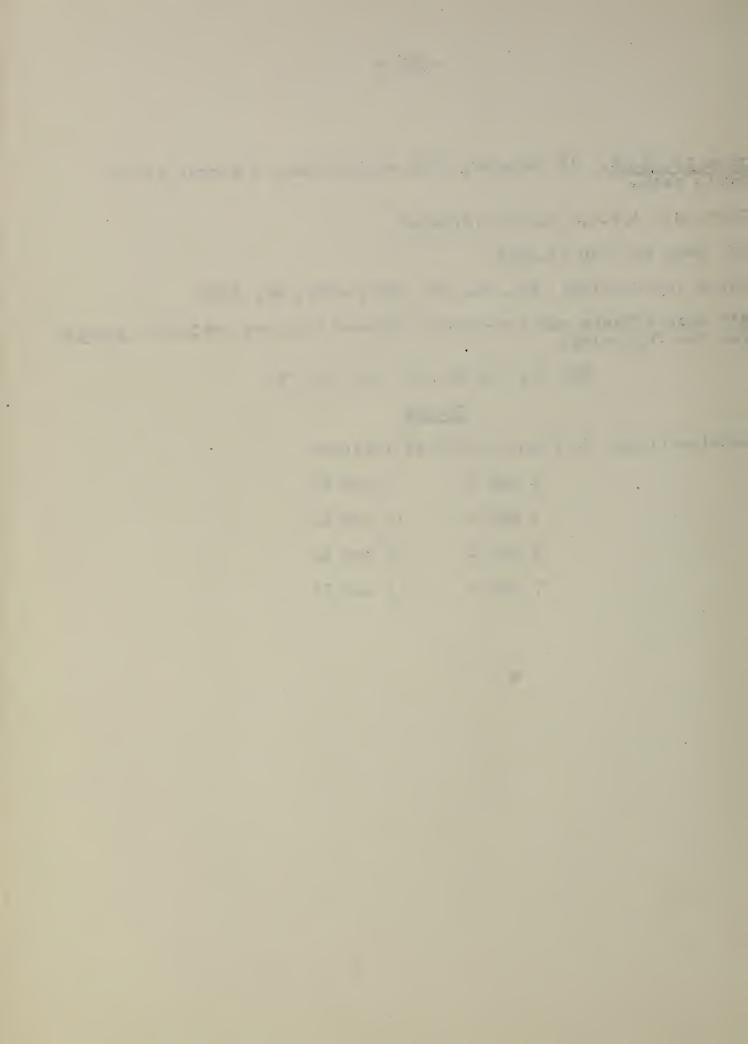
<u>Blocks</u>

Combine blocks in Plan 12.32.8 as follows:

1 and 2	9 and 10
3 and 4	11 and 12
5 and 6	13 and 14
7 and 8	15 and 16 .

<u>Plan 12.32.32.</u> 12 factors, 1/32 replication, 4 blocks of 32 units each.

Factors: A,B,C,D,E,F,G,H,J,K,L,M

I: Same as Plan 12.32.8

Block confounding: CD, ACEF, ADEF

All main effects and two factor interactions are estimable <u>except</u> for CD.

<center><u>Blocks</u></center>

Combine blocks in Plan 12.32.8 as follows:

<blockquote>
1, 2, 3, and 4

5, 6, 7, and 8

9, 10, 11, and 12

13, 14, 15, and 16 .
</blockquote>

THE NATIONAL BUREAU OF STANDARDS

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Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office.   Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.